Statistical Methods and Calibration in Finance and Insurance

MA6622 Assignment of Lectures 8-9

Exercises 1 and 2 are part of the assessment (Deadline: Tuesday 11th July).

1. Relevant information on the behaviour of an index is provided by its traded volume. Plot traded volumes for the HSI index in the following situations:

(a) Raw daily data for the last 6 months.

(b) Weekly means data for the last year.

(c) Using the Jarque-Bera statistics test whether you can assume normality for

(c1) Daily raw data $\{Vol(t)\}$,

(c2) Daily logarithmic data $\{\log Vol(t)\}$,

(c3) Weekly mean data $\{Week(t)\}$,

(c4) Weekly mean logarithmic data $\{\log Week(t)\}$,

Comment your resuls in terms of skewness and kurtosis.

2. (a) Plot the 2×2 cross-correlogram of the bidimensional time series that has in the first coordinate the HSI daily returns, and in the second one the traded volume. Use daily data from the last 6 months.

(b) Report the concurrent (contemporary) correlations, and the cross-correlations, for lags $h = 1, 2$ and $h = -1, -2$

(c) Do you think this information can be used in order to predict tomorrow's value of the HSI? Fundament your response.

3. Test whether the returns of the HSI are normally distributed, in the following situations:

- Daily data, one year data.
- Weekly data choosing the closing Friday quotation, and 50 last weeks.
- Weekly mean data for the same period as in the previous part.

- Monthly mean data, last 100 months.
- Quarterly mean data, from 1998 on.
- Yearly mean data, from 1998 on.

Take into account the sample size in order to decide which test to perform. (QQ-plot for small data sets, and Jarque-Bera for large data sets.)

4. In Excercise 3 of the assignement of Lectures 4-5, you where asked to choose a portfolio of three stocks, using daily data of the last three months.

(a) Test if this three dimensional data set can be considered mutivariate normally distributed.

(b) Plot the 3×3 cross correlogram of your porfolio returns. Do you find significative concurrent correlations and/or cross-correlations?

5. Currency risk exposure and multi-currency portfolios.

When considering multi-currency portfolios, in order to perform the statistical analysis, stock prices should be expressed in the same domestic currency (HKD in our case). The way to perform statistical analysis, is to convert into the domestic currency using the corresponding exchange rates of the date of quotation.

Observe that this multiplication introduces a new risk in your portfolio, due to exchange rate variations, that is called currency risk.

Taking this considerations into account, plot a correlogram (expresed in domestic currency) of three major indexes returns, the HSI, the S&P 500 and the FTSE 100 index.

6. We want to model the HSI weekly mean returns A and weekly mean traded volumes B through a bivariate autoregressive process of order 1 (VAR(1)).

(a) Test directly whether the return-volume series can be modelled by a white noise, with the Hosking Chi-Square test.

(b) Compute the estimator $\bar{\Phi}$ of the VAR(1) model for your series, and the corresponding bi-dimensional residuals. Check whether the eigenvalues of your estimation are greater than one.

(c) If the part (d) gives positive answer, test whether the residuals can be modelled by a white noise.

7. Given a random vector $\mathbf{X}$, a matrix $\mathbf{A}$ and a vector $\mathbf{b}$ prove that

$$\mathbf{E}(\mathbf{AX}+\mathbf{b}) = \mathbf{A}\,\mathbf{E}\,\mathbf{X}+\mathbf{b}, \qquad \mathbf{cov}(\mathbf{AX}+\mathbf{b}) = \mathbf{A}\,\mathbf{cov}(\mathbf{X})\mathbf{A}'.$$

8. Compute for a random variable $Z \sim \mathcal{N}(\mu, \sigma^2)$ the moments

$$\mathbf{E}(Z-\mu)^3, \quad \text{and} \quad \mathbf{E}(Z-\mu)^4,$$

to obtain that the skweness and kurtosis are

$$\gamma = \frac{\mathbf{E}(Z-\mu)^3}{\sigma^3} = 0, \quad \text{and} \quad \kappa = \frac{\mathbf{E}(Z-\mu)^4}{\sigma^4} - 3 = 0.$$

Hint: Use the representation $Z = \sigma X + \mu$ where X is a standard normal random variable (i.e. $X \sim \mathcal{N}(0,1)$) and use the values of the moments of X.