Statistical Methods and Calibration in Finance and Insurance

MA6622 Assignment of Lectures 6-7

Exercises 4,5,6,7 are part of the assessment (Deadline: Friday 7th July).

1. Find a pair of non-correlated random variables such that their squares are correlated. Can they be independent random varibles?

2. (a) Prove that if a strict stationary process has finite variance, then, all covariance are finite.
(b) Prove that a strict stationary process with finite variance is weak stationary.
(b) Show an example of a weak stationary process that is not strict stationary.

3. Determine whether the daily HSI returns have seasonal components or sistematical trend, in the daily data of last year.

4. Plot the correlogram for the HSI index returns, choosing an adequate number of data, in the following situations:

- Daily data for lags $h = 1, \ldots, 30$.
- Weekly data choosing the closing Friday quotation, and lags $h = 1, \ldots, 10$.
- Weekly mean data for the same period as the previous part.
- Monthly mean data, with lags $h = 1, \ldots, 12$.
- Yearly data, for lags $h = 1, 2, 3, 4$.

5. Plot the corresponding correlograms for the absolute value of the previous time series.

6. Test the hypothesis that the returns of the HSI index are strict white noise with the Ljung and Box test, with a confidence interval of $\alpha = 0.95$.

7. Suggest, based on visual analysis, whether the plotted data in the previous excercies can fit an AR(1) or MA(1) model. Choose the most feasible case, and perform the corresponding steps to fit the model.

8. Compute the autocorrelation function for a MA(1) time series. Compute the matrix $\mathbf{W}$ for a strict white noise.

9. Compute the autocorrelation function of a random walk, i.e. of a process of the form

$$Y(t) = \varepsilon(1) + \cdots + \varepsilon(t),$$

where $\{\varepsilon(t)\}$ is a strict white noise.

10. Construct with the Bartlett's formula confidence intervals with $\alpha = 0.05$ for the estimator of the correlations $\rho(1)$ and $\rho(2)$ in an AR(1) process.

11. Observe that for relatively big values of h given $(Z_1, \ldots, Z_h)$ independent standard gaussian random variables

$$J_h^2 = \sum_{i=1}^{h} (Z_i)^2 \sim \mathcal{N}(h, 2h)$$

Compare the $t_{0.95,30}$ value obtained through the χ^2 table with the one obtained with this approximation.

12. Check the autocovariance and autocorrelation functions for an MA(1) time series. Generalize for MA(q) time series.

13. What does "insider trading" mean? What is an "insider trader"?