Statistical Methods and Calibration in Finance and Insurance

MA6622 Assignment of Lectures 4-5

Exercises 2,3 and 4 are part of the assessment (Deadline: Friday 23th June).

1. Suppose, in a CAPM framework, that an asset A with return ρ_A has $\beta = 1$ with respect to a certain index of the market, with return ρ.
(a) Describe the relation between $\mathbf{E}\,\rho_A$ and $\mathbf{E}\,\rho$.
(b) Would you choose to invest in A or in the market index? Fundament your response.

2. Estimate the mean and the variance of the returns of the HSI index, using the last three months of daily data.

3. Suppose you want to invest 10.000 HKD in the Hong-Kong Stock Exchange, in three assets. Choose a portfolio of three stocks, evaluate the expected return and the variance of your portfolio (using daily data of the three last months) and compare your mean and return with the corresponding values of the HSI.

4. Estimate the corresponding β of the three stocks of the previous excercise.

5. Read the clipping from Financial Times May 29th 2006. (Download the file *financial-times.pdf* at the course website.)

6. (a) Find the maximum possible value of $ar_A + br_B$, where $0 < r_A < r_B$ are given, $a \geq 0, b \geq 0$ and $a + b = 1$. (b) Find the minimum possible value of $a^2\sigma_A^2 + b^2\sigma_B^2$ where $\sigma_A > 0$ and $\sigma_B > 0$ are given, and $a \geq 0, b \geq 0$ and $a + b = 1$.

7. Assume that you invest in two noncorrelated assets A and B, with expected returns $r_A < r_B$, and variances σ_A, σ_B.
(a) Find the portfolio with maximum possible return.
(b) Find the portfolio with minimum possible variance.

8. Consider two assets A, B with (in annual %)

$$\begin{array}{ccc} \bar{x}_A = 10 & \bar{x}_B = 5 & \bar{x}_C = 3 \\ \bar{\sigma}_A = 8 & \bar{\sigma}_B = 10 & \bar{\sigma}_C = 12 \\ \bar{c}_{AB} = 0 & \bar{c}_{AC} = 0 & \bar{\sigma}_{BC} = 0 \end{array}$$

The expected return of a portfolio with proportions $(1/3, 1/3, 1/3)$ is 6, with a standard deviation 5.85. Find the efficient portfolio with maximum expected return for this standard deviation.

9. Assume that $X, Y, X_1, \dots, X_n$ are random variables. Denote

$$\mathbf{var}\, X = \mathbf{E}\left(X - \mathbf{E}\, X\right)^2,$$
$$\mathbf{cov}(X,Y) = \mathbf{E}\left(X - \mathbf{E}\, X\right)\left(Y - \mathbf{E}\, Y\right) = \sqrt{\mathbf{var}\, X\, \mathbf{var}\, Y}\, \rho(X,Y).$$

Prove the following formulas.
(a)$\mathbf{var}(aX) = a^2\, \mathbf{var}\, X$.
(b) $\mathbf{var}\, X = \mathbf{E}(X^2) - (\mathbf{E}\, X)^2$.
(c) $\mathbf{cov}(X,Y) = \mathbf{E}(XY) - \mathbf{E}\, X\, \mathbf{E}\, Y$
(d)

$$\mathbf{var}(X_1 + \cdots + X_n) = \sum_{k=1}^{n} \mathbf{var}\, X_k + 2 \sum_{1 \le j < k \le n} \mathbf{cov}(X_j, X_k).$$

(e) Remember that if X, Y are independent, then $\mathbf{E}(XY) = \mathbf{E}\, X\, \mathbf{E}\, Y$. Prove that if X, Y are independent, then $\mathbf{cov}(X,Y) = \rho(X,Y) = 0$.
(f) Assuming that $X_1, \dots, X_n$ are independent prove that

$$\mathbf{var}(X_1 + \cdots + X_n) = \sum_{k=1}^{n} \mathbf{var}\, X_k.$$