

Statistical Methods and Calibration in Finance and Actuarial Science

MA6622

Assignment of Lectures 22 to 25

1. Prove that the duration of a coupon bearing bond can not be larger than its time to maturity.
2. Prove, using Itô formula, that the stochastic process

$$r(t) = \frac{\theta}{\alpha} + e^{-\alpha t} \left[r_0 - \frac{\theta}{\alpha} \right] + \sigma e^{-\alpha t} \int_0^t e^{\alpha s} dW(s)$$

satisfies Vasicek diffusion equation

$$dr(t) = (\theta - \alpha r(t)) dt + \sigma dW(t).$$

Based on the first formula, prove that $r(t)$ is a gaussian random variable, with

$$\begin{aligned} \mathbf{E} r(t) &= \frac{\theta}{\alpha} + e^{-\alpha t}, \\ \mathbf{var} r(t) &= \frac{\sigma^2}{2\alpha} [1 - e^{-2\alpha t}]. \end{aligned}$$

Remember that, for a deterministic function $f(s)$

$$\mathbf{var} \int_0^t f(s) dW(s) = \int_0^t f(s)^2 ds.$$