The distribution of the maximum of a Lévy process with positive jumps of phase-type *

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Abstract

Consider a Lévy process with finite intensity positive jumps of the phase-type and arbitrary negative jumps. Assume that the process either is killed at a constant rate or drifts to $-\infty$. We show that the distribution of the overall maximum of this process is also of phase-type, and find the distribution of this random variable. Previous results (hyperexponential positive jumps) are obtained as a particular case.

Keywords: Lévy processes, Phase-type distributions, maximum of a Lévy process, hyperexponential distributions, ruin probabilities.

1 Introduction and main result

Consider a Lévy process $X = \{X_t\}_{t\geq 0}$ with finite intensity positive jumps of the phase-type and arbitrary negative jumps. In order to describe the Lévy jump measure of X consider a random variable U of phase-type, with distribution $B(y)$ and representation $(\pi, \mathbf{T}, d)$, where d is a positive integer, $\pi = (\pi_1, \dots, \pi_d)$ is the initial probability distribution, and the intensity matrix $\mathbf{T}$, and asociated exit rates vector $\mathbf{t}$, are given by

$$\mathbf{T} = \begin{bmatrix} t_{11} & \cdots & t_{1d} \\ \vdots & \vdots & \vdots \\ t_{d1} & \cdots & t_{dd} \end{bmatrix}, \qquad \mathbf{t} = \begin{bmatrix} t_1 \\ \vdots \\ t_d \end{bmatrix}, \tag{1}$$

where $\mathbf{t}$ satisfies $t_j + \sum_{k=1}^{d} t_{jk} = 0$ $(j = 1, \dots, d)$. For details and general results on phase-type distributions we refer to Asmussen (2000), and Asmussen (1992).

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(In particular, we borrow the notation for the phase-type distributions from Asmussen (1992).)

The Lévy measure of the considered Lévy process is

$$K(dy) = \begin{cases} \lambda b(y), & y > 0, \\ K^-(dy), & y < 0, \end{cases} \tag{2}$$

where $b(y) = B'(y) = \pi \exp(\mathbf{T}y)\mathbf{t}$ is the density of the random variable U, the constant $\lambda > 0$ is the intensity of positive jumps, and $K^-(dy)$ is an arbitrary Lévy measure supported on the set $(-\infty, 0)$. For general references on Lévy process we refer to Bertoin (1996) and Sato (1999).

The purpose of this paper is, given $\gamma \geq 0$, to compute the distribution of the random variable

$$M = \max_{0 \leq t < T(\gamma)} X_t, \tag{3}$$

where $T(\gamma)$ is an exponential random variable with parameter $\gamma > 0$, independent of X, and we set $T(0) = \infty$. We always assume that the random variable M is proper, this always happens when $\gamma > 0$, and follows form the fact (that we assume) that the process drifts to $-\infty$, i.e. $\mathbf{P}(\lim_{t\to\infty} X_t = -\infty) = 1$, when $\gamma = 0$.

The characteristic exponent of X, defined by $\Psi(z) = \log \mathbf{E}\, e^{zX_1}$ for the complex values of z such that the last expectation is finite, is given, by the Lévy-Khinchine formula, by

$$\Psi(z) = az + \frac{1}{2}\sigma^2 z^2 + \int_{-\infty}^{0} (e^{zy} - 1 - zh(y))K^-(dy) + \lambda(\mathbf{E}\, e^{zU} - 1), \tag{4}$$

where the real constant a is the drift, fixed once the truncation function $h(y) = y\mathbf{1}_{\{-1<y<0\}}$ is fixed, and $\sigma \geq 0$ is the variance of the gaussian part of X.

The last summand in (4), that we denote $\Psi^+(z)$, is

$$\Psi^+(z) = \lambda(\mathbf{E}\, e^{zU} - 1) = \lambda\big(\pi(-z\mathbf{I} - \mathbf{T})^{-1}\mathbf{t} - 1\big), \tag{5}$$

where $\mathbf{I}$ is the $d \times d$ identity matrix. Defining $\Psi^-(z) = \Psi(z) - \Psi^+(z)$ we can represent the process X as the sum of two independent Lévy processes, i.e. $X_t = X_t^+ + X_t^-$ $(t \geq 0)$ where $X^+ = \{X_t^+\}_{t\geq 0}$ is a Lévy process with characteristic exponent $\Psi^+(z)$; $X^- = \{X_t^-\}_{t\geq 0}$ is a Lévy process with characteristic exponent $\Psi^-(z)$.

In order to describe the distribution of M in (3) we must distinguish the following two cases:

(A) The process X^- is not the negative of a subordinator. (This is the case, for instance, when $\sigma > 0$.)

(B) The process X^- is the negative of a subordinator, including a deterministic negative drift.

We begin by case (A). The idea is to show that M in (3) can be obtained as the maximum of an associated random walk, with increments that are the difference of two independent random variables, the first one of the phase-type ($-\infty$ modified when $\gamma > 0$), and to apply the clasical principles of Pollachek-Khinchine renewal theory as presented in Chapter VI in Asmussen (2000).

Introduce the epoch of positive jumps of X by

$$T_0 = 0, \qquad T_{n+1} = \inf\{t > T_n \colon \Delta X_t > 0\} \quad (n = 0, 1, \dots),$$

and denote

$$T_n^\gamma = T_n \wedge T(\gamma), \qquad T_n^0 = T_n \quad (n = 1, 2, \dots).$$

The random variables $T_{n+1} - T_n$ $(n = 0, 1, \dots)$ are independent and identically distributed, with exponential distribution with parameter λ.

Introduce

$$U_0 = \max_{0 \leq t < T_1^\gamma} X_t = \max_{0 \leq t < T_1^\gamma} X_t^- . \tag{6}$$

This random variable is the maximum of a Lévy process with no positive jumps (the process X^-) up to an independent random time T_1^γ, with exponential distribution with parameter $\lambda + \gamma$. In consequence, U_0 has an exponential distribution with parameter p, the positive root of $\Psi^-(z) = \lambda + \gamma$ (see Corollary VII.1.2 in Bertoin (1996)).

Denote now the $-\infty$ modified (when $q > 0$) magnitude of the positive jumps of the process X by

$$U_n = \begin{cases} \max\{X_t - X_{(T_n)-} \colon t \in [T_n, T_{n+1}^\gamma)\}, & \text{if } T_n < T(\gamma), \\ -\infty, & \text{if } T(\gamma) < T_n. \end{cases}$$

For $n \geq 1$, on the event $\{T_n < T(\gamma)\}$ the random variable U_n is of phase-type, as

$$U_n = X_{T_n} - X_{(T_n)-} + \max_{T_n \leq t < T_{n+1}^\gamma} (X_t - X_{(T_n)}),$$

is the sum of two independent phase-type distributed random variables, the first with distribution $B(y)$, and the second distributed as U_0 in (6). In conclusion, conditional to the event $\{T_{n-1} < T(\gamma)\}$, the random variable U_n is defective with probability $\gamma/(\lambda + \gamma)$, has initial distribution

$$\mu = \Big(\frac{\lambda \pi_1}{\lambda + \gamma}, \dots, \frac{\lambda \pi_d}{\lambda + \gamma}, 0\Big),$$

and representation $(\mu, \mathbf{S}, d + 1)$, where the intensity matrix $\mathbf{S}$, with the corresponding exit rates vector $\mathbf{s}$, are given by

$$\mathbf{S} = \begin{bmatrix} t_{11} & \cdots & t_{1d} & t_1 \\ \vdots & \vdots & \vdots & \vdots \\ t_{d1} & \cdots & t_{dd} & t_d \\ 0 & \cdots & 0 & -p \end{bmatrix}, \qquad \mathbf{s} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ p \end{bmatrix}. \tag{7}$$

The absorbing state, for convinience, is $\Delta = -\infty$.

Introduce now the magnitude of negative increments by

$$V_n = X_{(T_n)-} + U_n - X_{(T_{n+1})-} \qquad (n = 1, 2, \dots).$$

The random variables V_n are independent and identically distributed, with distribution $A(y)$, and characteristic function

$$\begin{aligned}\hat{A}(z) &= \mathbf{E}\, e^{zV_1} = \mathbf{E}\, e^{-z(-V_1)} = \mathbf{E} \exp(-z \inf_{0 \le t \le T_1^\gamma} X_t^-) \\ &= \mathbf{E}\, e^{-zX^-_{T_1^\gamma}} \left(\mathbf{E}\, e^{-zU_0}\right)^{-1} \\ &= \frac{\lambda + \gamma}{\lambda + \gamma - \Psi^-(-z)} \times \frac{p+z}{p},\end{aligned}$$

where we rely on fluctuation identities and Rogozin's (1966) factorization identity. (See Bertoin (1996, chapter VI).)

Consider finally the sequence $U_0, V_1, U_1, V_2, U_2, \dots$, and set $V_0 = 0$. Based on the independence of increments of the Lévy process, we obtain the independence of the vectors $(V_0, U_0), (V_1, U_1), (V_2, U_2), \dots$. On the other side, on each interval of the form $[T_n, T_{n+1})$, the path decomposition of the process (as described in Theorem VI.2.5 of Bertoin (1996)) gives that each pair V_n, U_n is formed by independent random variables. In conclussion, the consiedered sequence is formed by mutually independent random variables.

In view of our construction, we have

$$M = \max_{0 \le t \le T(\gamma)} X_t = \max_{n \ge 0} \sum_{k=0}^{n} (U_k - V_k) = U_0 + \Big(\max_{n\ge 1} \sum_{k=1}^{n} (U_k - V_k)\Big)^+.$$

In order to find the ladder heigt distribution of the random walk $\sum_{k=1}^{n}(U_k - V_k)$, we apply the Proposition VIII.4.3 in Assmussen (2000) (take conditional probability on the event $\{V_1 = y\}$ integrating with respect to $A(dy)$) to obtain that the ladder heigth is of phase-type, with reprentation $(\mu+, \mathbf{S}, d+1)$, where $\mu+$ is the solution of the nonlinear vectorial equation

$$\mu+ = \mu \int_0^\infty \exp\big((\mathbf{S} + \mathbf{s}\mu+)y\big) A(dy) = \mu \hat{A}(\mathbf{S} + \mathbf{s}\mu+). \tag{8}$$

The maximum of the random walk $\big(\max_{n\ge 1} \sum_{k=1}^{n}(U_k - V_k)\big)^+$ is the geometric compound of the ladder heights $(G_k)_{k\ge 1}$, each of one has distribution of the phase-type with representation $(\mu+, \mathbf{S}, d+1)$, and in consequence, we have

$$M = U_0 + G_1 + \cdots + G_N$$

where N is a geometric random variable, independent of the random walk, with $P(N = 0) = 1 - \|\mu+\|$. As the random variable U_0 can be represented by $(\mathbf{e}_{d+1}, \mathbf{S}, d+1)$, where

$$\mathbf{e}_{d+1} = (0, \dots, 0, 1), \tag{9}$$

we obtain that M is a geometric compound of random variables of phase-type, with the same intensity matrix $\mathbf{S}$, and the first has a different initial vector $\mathbf{e}_{d+1}$. By example A5.10 in Asmussen (2000), we obtain that the maximum M has representation $(\mathbf{e}_{d+1}, \mathbf{S} + \mathbf{s}\mu+, d+1)$.

Consider now case (B). The situation is simpler, and in particular, we do not have to rely on fluctuation results, reducing the problem to the computation of the distribution of a random walk more directly. Following our previous developement, we see that the differences are

- With the same definition, $U_0 = 0$.
- With the same definition, for $n \geq 1$, we have $U_n = X_{T_n} - X_{(T_n)-}$, and $\mathbf{S} = \mathbf{T}$. The corresponding initial distribution for U_n is

$$\mu = \Big(\frac{\lambda}{\lambda+\gamma}\pi_1, \dots, \frac{\lambda}{\lambda+\gamma}\pi_d\Big).$$

- The random variables V_n are defined through the same formula, but

$$\hat{A}(z) = \mathbf{E}\, e^{zV_1} = \frac{\lambda+\gamma}{\lambda+\gamma-\psi^-(z)},$$

  being equivalent to the case $p = \infty$ in the corresponding formula for case (A).
- The initial distribution of the ladder heights is $\pi+$, defined as the solution to the equation

$$\pi+ = \frac{\lambda}{\lambda+\gamma}\pi\int_0^\infty \exp\big((\mathbf{T} + \mathbf{t}\pi+)y\big)A(dy) = \frac{\lambda}{\lambda+\gamma}\pi\hat{A}(\mathbf{S} + \mathbf{s}\mu+). \quad (10)$$

In conclusion, in this case

$$M = G_1 + \cdots + G_N$$

and the distribution of M has representation $(\pi+, \mathbf{T} + \mathbf{t}\pi+, d)$.

We have then proved the following result.

Theorem 1. *Consider a Lévy process X with characteristic exponent $\Psi(z)$ defined in* (4) *and* (5)*, a constant $\gamma \geq 0$. Let $T(\gamma)$ be an exponential random variable independent of X with parameter $\gamma > 0$, let $T(0) = 0$, and define $M = \max\{X_t : 0 \leq t < T(\gamma)\}$. Then:*

- *In case* (A)*, the random variable M is of phase-type, with a representation $(\mathbf{e}_{d+1}, \mathbf{S} + \mathbf{s}\mu+, d+1)$ defined, respectively in* (9)*,* (7)*, and* (8)*.*
- *In case* (B)*, the random variable M is of phase-type, whith a representation $(\pi+, \mathbf{T} + \mathbf{t}\pi+, d)$ defined, respectively, in* (1) *and* (10)*.*

2 Hyperexponential positive jumps

Assume in this section that positive jumps in (2) are distributed according to a mixture of exponential distributions, i.e. an hyperexponential distribution, with a density

$$b(y) = \sum_{k=1}^{d} a_k \alpha_k e^{-\alpha_k y}, \qquad y > 0,$$

where $0 < \alpha_1 < \cdots < \alpha_d$, the positive mixture coefficients $a_1, \dots, a_k$ satisfy $a_1 + \cdots + a_d = 1$; and $K^-(dy)$ remains arbitrary. The characteristic exponent in (4), in this particular case, is

$$\Psi(z) = az + \frac{1}{2}\sigma^2 z^2 + \int_{-\infty}^{0} \big(e^{zy} - 1 - zh(y)\big) K^-(dy) + \lambda \sum_{k=1}^{d} a_k \frac{z}{\alpha_k - z} \tag{11}$$

In case (A), the equation $\Psi(z) = \gamma$ has $d + 1$ positive roots ρ_k, that satisfy

$$0 < \rho_1 < \alpha_1 < \rho_2 < \cdots < \alpha_d < \rho_d + 1.$$

This is clear in the case $\gamma > 0$, and follows from the fact that $\Psi'(0+) > 0$ (that is equivalent to $\lim_{t\to\infty} X_t = -\infty$ *a.s.*) in case $\gamma = 0$ (see details in Mordecki (1999)).

The characteristic function of the increment of the associated random walk, is

$$\begin{aligned}\hat{F}(z) = \mathbf{E}\exp(U_1 - V_1) &= \frac{\lambda}{\lambda+\gamma} \mathbf{E}\, e^{zU} \frac{\lambda+\gamma}{\lambda+\gamma-\psi^-(z)} \\ &= \frac{\Psi^+(z)+\lambda}{\lambda+\gamma-\Psi^-(z)}.\end{aligned}$$

In consequence, the equation $\hat{F}(z) = 1$ has the same $d+1$ real and positive roots, as the equation $\Psi(z) = \gamma$, and from Corollary VIII.4.6 in Asmussen (2000) we obtain that the matrix $\mathbf{S} + \mathbf{s}\mu+$ has $-\rho_1, \dots, -\rho_{d+1}$ as eigenvalues.

Let us now, with this information, compute the characteristic function of M. Denoting the unknown vector $\mu+ = (x_1, \dots, x_{d+1})$, $\mathbf{Q} = \mathbf{S} + \mathbf{s}\mu+$, and $\mathbf{q}$ the corresponding exit rates vector, we have

$$\mathbf{Q} = \begin{bmatrix} -\alpha_1 & \cdots & 0 & \alpha_1 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & -\alpha_d & \alpha_d \\ x_1 p & \cdots & x_d p & (x_{d+1}-1)p \end{bmatrix} \qquad \mathbf{q} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ p(x_1 + \cdots + x_{d+1} - 1) \end{bmatrix}.$$

According to (c) in Theorem VIII.1.5 in Asmussen (2000)

$$\mathbf{E}\, e^{zM} = \mathbf{e}_{d+1}(-zI - \mathbf{Q})^{-1}\mathbf{q}. \tag{12}$$

Taking into account the presence of zeros in $\mathbf{e}_{d+1}$ and $\mathbf{q}$, we must compute only one element of the inverse matrix in (12). As we know the eigenvalues of $\mathbf{Q}$ we have $\det(\mathbf{Q}-zI)=\prod_{k=1}^{d+1}(-\rho_k-z)$, and $\det(-\mathbf{Q}-zI)=(-1)^{d+1}\det(\mathbf{Q}+zI)=\prod_{k=1}^{d+1}(z-\rho_k)$. Appliyng Cramer's rule

$$\mathbf{E}\,e^{zM}=p\big(\|\mu+\|-1\big)\times\frac{\prod_{k=1}^{n}(z-\alpha_k)}{\prod_{k=1}^{d+1}(z-\rho_k)}.$$

Evaluating this expresion an $z=0$, we conclude that

$$\begin{aligned}\mathbf{E}\,e^{zM}&=\frac{\prod_{k=1}^{d}(z-\alpha_k)}{\prod_{k=1}^{d+1}(z-\rho_k)}\times\frac{\prod_{k=1}^{d}(-\alpha_k)}{\prod_{k=1}^{d+1}(-\rho_k)}\\&=\frac{\prod_{k=1}^{d}(1-z/\alpha_k)}{\prod_{k=1}^{d+1}(1-z/\rho_k)}=\sum_{k=1}^{d+1}A_k\frac{\rho_k}{\rho_k-z},\end{aligned}\tag{13}$$

where we find the coeficients $A_1,\dots,A_{d+1}$ introduced in the last expression through the simple fractional expansion Theorem, to obtain

$$A_j=\frac{\prod_{k=1}^{d}(1-\rho_j/\alpha_k)}{\prod_{k=1,k\neq j}^{d+1}(1-\rho_j/\rho_k)}\qquad(j=1,\dots,d+1).$$

This is the result in Mordecki (1999).

Consider now case (B). As X^- is the negative of a subordinator, based on Bertoin (1996, pp. 72–73), we obtain that

$$\Psi^-(z)=az+\int_{-\infty}^{0}(e^{zy}-1)K^-(dy),$$

where $a\le 0$, and $\int_{-\infty}^{0}\big(1\wedge|y|\big)K^-(dy)<\infty$, i.e. the process X^- has finite variation and negative drift. Furthermore, $\lim_{x\to\infty}\Psi^-(x)=a$ (x real). Based on this considerations, and on the fact that the function $\Psi^-(x)$ is convex, we obtain that the equation

$$\Psi(z)=az+\int_{-\infty}^{0}(e^{zy}-1)K^-(dy)+\lambda\sum_{k=1}^{d}a_k\frac{z}{\alpha_k-z}=\gamma\tag{14}$$

has d real and positive roots ρ_k $(k=1,\dots,d)$ that satisfy

$$0<\rho_1<\alpha_1<\cdots<\rho_d<\alpha_d.\tag{15}$$

This is again clear in the case $\gamma>0$, and follows from the fact that $\Psi'(0+)>0$ (that is equivalent to $\lim_{t\to\infty}X_t=-\infty$ $a.s.$) in case $\gamma=0$.

We now consider the ε-perturbed Lévy process $X^\varepsilon=\{X_t^\varepsilon\}_{t\ge 0}$ with characteristic exponent

$$\Psi^\varepsilon(z)=az+\frac{1}{2}\varepsilon^2z^2+\int_{-\infty}^{0}(e^{zy}-1)K^-(dy)+\lambda\sum_{k=1}^{d}a_k\frac{z}{\alpha_k-z},$$

that satisfies the hypothesis of case (A) in this section, and has $d+1$ roots ρ_k^ε $(k=1,\dots,d+1)$, that satisfy the conditions

$$0<\rho_1^\varepsilon<\alpha_1<\rho_2^\varepsilon<\cdots<\alpha_d<\rho_{d+1}^\varepsilon.$$

As $\varepsilon\to 0$, we have $\rho_k^\varepsilon\to\rho_k$ $(k=1,\dots,d)$ in (15), and $\rho_{d+1}^\varepsilon\to\infty$. As $\Psi^\varepsilon(z)\to\Psi(z)$ in (14), we have weak convergence of processes, and in consequence,

$$M^\varepsilon=\max_{0\le t<T(\gamma)}X_t^\varepsilon\to M=\max_{0\le t<T(\gamma)}X_t \qquad \text{(weakly)}.$$

The characteristic function of M can then be obtained taking limit in (13), so

$$\begin{aligned}\mathbf{E}\,e^{zM}&=\lim_{\varepsilon\to 0}\mathbf{E}\,e^{zM^\varepsilon}=\lim_{\varepsilon\to 0}\frac{\prod_{k=1}^{d}(1-z/\alpha_k)}{\prod_{k=1}^{d+1}(1-z/\rho_k^\varepsilon)}\\ &=\frac{\prod_{k=1}^{d}(1-z/\alpha_k)}{\prod_{k=1}^{d}(1-z/\rho_k)}=\sum_{k=1}^{d}A_k\frac{\rho_k}{\rho_k-z},\end{aligned}\tag{16}$$

where, in the last step, we again apply the simple fractional expansion Theorem, to obtain

$$A_j=\frac{\prod_{k=1}^{d}(1-\rho_j/\alpha_k)}{\prod_{k=1,k\neq j}^{d}(1-\rho_j/\rho_k)}\qquad (j=1,\dots,d).$$

This concludes the proof of the following result.

Theorem 2. *Let X be a Lévy process with finite intensity hyperexponential positive jumps, and characteristic exponent given by* (11). *Consider $\gamma\ge 0$ and $T(\gamma)$ an exponential random variable independent of X with parameter $\gamma>0$; set $T(0)=0$, and define $M=\max\{X_t\colon 0\le t<T(\gamma)\}$. Then:*

- *In case* (A)*, the random variable M is also hyperexponential, with $d+1$ components, and characteristic function given in* (13).
- *In case* (B)*, the random variable M is also hyperexponential, now with d components, and characteristic function given in* (16).

3 Conclussions

In the presented paper we consider the problem of the determination of the distribution of the overall maximum of a Lévy process $X=\{X_t\}_{t\ge 0}$, with characteristic exponent $\Psi(z)$, either killed at a constant rate $\gamma>0$ or that drifts to $-\infty$. As follows from the corresponding results obtained for random walks (see for instance Assmusen (1992) or Kemperman (1961)) it seems that the most general framework to obtain closed explicit solutions in the posed problem, is when one has arbitrary negative jumps for X, and a specification on the structure of positive jumps.

In what respects to the methods available to solve this problem, we can indicate at least three:

(1) The Wiener-Hopf factorization method, based in Rogozin (1966) factorization identity, that states

$$\frac{\gamma}{\gamma - \Psi(z)} = \mathbf{E}\, e^{zM}\, \mathbf{E}\, e^{zI}, \tag{17}$$

where $M = \max\{X_t \colon 0 \le t < T(\gamma)\}$, $I = \min\{X_t \colon 0 \le t < T(\gamma)\}$, and $T(\gamma)$ is an exponential time (with parameter $\gamma > 0$) independent of X. The uniqueness of the factors in (17) allows to identify the characteristic functions of M and I, in different situations. For instance, Asmussen et al. (2002) gave the distribution of M when positive and negative jumps are of phase-type. This factorization seems to be aplicable when the Lévy measure has a rational Laplace transform.

(2) The martingale method. Once the form of the distribution is known, the method consists in the application of Doob's optional sampling theorem (frecuently with the help of Ito's formula) to verify that the given form is efectively the distribution to find. This method was used in Mordecki (1999) to obtain the distribution in case of a Lévy process with arbitrary negative jumps and hyperexponential positive jumps; and in Asmussen et. al (2002) an outline of the proof is given when considering the more general situation, of positive jumps of phase-type and arbitrary negative jumps.

(3) The Pollachek-Khinchine method, based on renewal arguments, as exposed in Asmussen (2000) for random walks with phase-type positive jumps, adapted in the presented article to the case of Lévy processes, to consider the case when positive jumps are of phase-type and negative jumps remain arbitrary.

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References

[1] Asmussen, S. Phase-type representations in random walk and queueing problems. The Annals of Probability (1992) Vol. 20, No. 2, 772–789

[2] Asmussen, S. Ruin Probabilities. World Scientific, Singapore, 2000.

[3] S. Asmussen, F. Avram and M. Pistorius. Russian and American put options under exponential phase-type Lévy models. Preprint 1240, Department of Mathematics, Utrecht University, 2002, pp. 34.

[4] Bertoin J., Lévy Processes. Cambridge University Press, Cambridge, 1996.

[5] Kemperman, J.H.B., The passage problem for a stationary Markov chain. The University of Chicago Press, Chicago, 1961.

[6] Mordecki E. Ruin probabilities for a Lévy process with mixed exponential negative jumps. Pre-Mat 28, (1999) (To appear in Theory of Probability and its Applications)

[7] Rogozin, B.A.(1966). On distributions of functionals related to boundary problems for processes with independent increments. Theory of Probability and its Applications, Vol. XI, 4, 580–591.

[8] Sato, Ken-iti. Lévy processes and infinitely divisible distributions. Cambridge Studies in Advanced Mathematics, 68. Cambridge University Press, Cambridge, 1999.