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PRICING BOND OPTIONS IN EMERGING MARKETS: A CASE STUDY

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Abstract. We propose two methodologies to price sovereign bond options in emerging markets. The motivation is to provide hedging protection against price fluctuations, departing from the not liquid data provided by the stock exchange. Taking this into account, we first compute prices provided by the Jamshidian formula, when modeling the interest rate through Vasicek model, with parameters estimated with the help of the Kalman filter. The second methodology is the pricing strategy provided by the Black-Derman-Toy tree model. A numerical comparison is carried out. The first equilibrium approach provides parsimonious modeling, is less sensitive to daily changes and more robust, while the second non-arbitrage approach provides more fluctuating but also what can be considered more accurate option prices.

1. **Introduction.** In many emerging markets the bond hedging instruments are poorly developed, exposing the bond's holders to severe risks, specially taking into account the dependence structure of an interconnected international financial system. After the 2008-2009 international crises, the great liquidity present in international markets induced a change of direction in capital fluxes, with a significant increase of emerging markets capital flows. Regarding derivatives, only 10% of financial derivatives have underlyings from emerging markets, the contracts used being the simplest ones. The underlying of these derivatives is mainly a currency, due to the relevance of the currency risk present in the emerging economies. For general reference see the publications of the Bank for International Settlements, as [2] or [19]. In particular, in Latin America, there exists currency derivatives in almost all countries. Regarding interest rate, derivatives are scarce, with the exception of Brazil. In Uruguay, the regulated currency derivatives have increased significantly, from 252 millions USD in 2011 to 1345 millions USD in 2015, having no regulated interest rate derivatives.

The purpose of this work is then to provide an adequate methodology for pricing European bond options in emerging markets. The lack of liquidity entails the need to use a large amount of information, i.e. the prices of practically *all* bonds available

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in the market. Furthermore, as we have no derivatives, we have no equivalent risk-neutral pricing measure. Although bond prices itself have information about the future behavior of the market, in developed markets, available derivatives (as for instance credit default swaps) are used to obtain the risk-neutral measure. More precisely, in order to price options, we take as an example the Uruguayan bond market, considering two situations: USD nominated and inflation-indexed bond options, that for short we refer as USD and UI bond options respectively. Here UI stands for inflation-indexed ("unidades indexadas").

According to a report issued by the Ministry of Economy and Finance of Uruguay in October 2016, the total amount of the Uruguayan debt was 26087 millions of USD in September 2016. This debt was mainly (92%) composed by bonds issued by the government. Only the 26% of this debt is under Uruguayan jurisdiction, the remaining debt being mainly under USA jurisdiction (i.e. corresponds to bonds issued by the Uruguayan government in USD markets and through USA financial institutions). It is important to notice that the 46% of the total debt is nominated in the local currency (UYU pesos), meanwhile the remaining 54% is nominated in foreign currencies (USD, Euros and Yens). It should be mentioned that this 54%-46% of foreign vs. local currency debt is the result of a Central Bank of Uruguay ("Banco Central del Uruguay" - BCU) policy to un-dollarize the debt: in 2004 this proportion was 89%-11% (foreign vs. local currencies). Regarding the holders of this debt, the main participants of the local bond markets are the pension funds (Administradoras de Fondos de Ahorro Provisional - AFAPs) created by law in the country in 1996. As a result of legal regulation, these institutions, according to data of 2016, hold a portfolio of more than 60% invested in Uruguayan bonds, from a total turnover of 12483 millions of USD. In short, approximately the 30% of the Uruguayan bonds integrate the AFAP's portfolios (for details see [17]).

As an example of possible hedging uses of derivatives in the bond maket, in mid 2013, as a consequence of a FED announcement, the yield curve of inflation-indexed bonds estimated by the Electronic Stock Exchange of Uruguay ("Bolsa electrónica de valores Sociedad Anónima" - BEVSA) had a significant increase, corresponding to an important drop in all the bond prices. For instance, the movement in the 10 year UI yield bond, was larger than 2%. This situation motivated a large loss for corporate investment institutions, as pension funds, with a cost of approximately the 5% of the total portfolio. This type of movements in the yield curve induce undesired volatility with negative consequences in important areas as the social security, motivating the idea of the introduction of hedging derivatives into the bond market. Figure 1 shows the consequences of this drop in terms of the average monthly returns of the year.

The rest of the paper is as follows. In Section 2 we introduce our two models and explain the calibration strategies employed. In Section 3 we describe the data used to calibrate the models, and present the obtained option prices, both for USD and UI bonds. In Section 4 we present some conclusions.

2. **Models for bond yield and options.**

The development of stochastic bond pricing models has an initial milestone in the 1977 work of Vasicek [20], where the instantaneous interest rate is considered as the solution of a mean-reverting stochastic differential equation, based on equilibrium considerations. Posterior contributions are based on free-arbitrage considerations, and constitute a second generation of pricing bond models, producing a term structure of interest rates that fits the

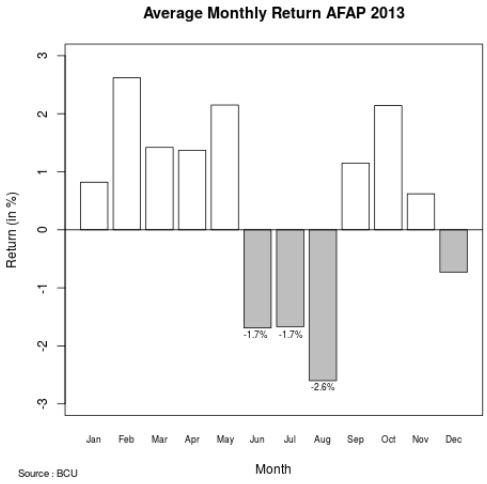


FIGURE 1. Monthly returns of AFAPS. Year 2013 (in UYU).

whole initial bond price curve. Let us mention in this context the 1992 contribution of Heath-Jarrow-Morton [11]. Other examples of free-arbitrage pricing are the papers by Ho and Lee [12], Hull and White [13], Black, Derman and Toy [3], Black and Karasinski [4]. For a general reference on the subject see, within others, the books by Brigo and Mercurio [5] or Filipovic [9].

Due to the fact that in Uruguay the bond derivatives market is not developed, we propose two different pricing methodologies. We first compute reference prices in what can be considered a benchmark pricing strategy, modeling the interest rate through Vasicek model, calibrated with the help of the Kalman filter, and apply Jamshidian option valuation formula [15]. We choose a one factor model, due to simplicity and to the existence of a closed formula for option prices. Models with more factors, as in [18], can also be considered. This procedure is robust because it models the complete interest rate structure with the help of only three parameters. It provides then general reference prices. Nevertheless, it does not have the flexibility to capture the complete initial yield curve. Although this fact theoretically provides arbitrage opportunities, in a non liquid market transactions costs are so high that this opportunities can not be cashed. The second methodology that we use is the one proposed by Black, Derman and Toy [3]. It consists in the calibration of a tree, with two parameters for each day (that can be considered as daily drift and volatility). Due to its flexibility and theoretical basis, it provides non-arbitrage bond option prices. Option prices are computed with a backwards algorithm.

2.1. **The Vasicek model.** Vasicek [20] proposes a model for the short rate through a stochastic differential equation driven by a Wiener process,

$$dr_t = a(b - r_t)dt + \sigma dW_t, \qquad r(0) = r_0, \tag{1}$$

where a, b and σ are positive constants and $\{W_t\}$ is a standard Wiener process defined in a stochastic basis $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathbf{P})$ (see [20]). The solution to this equation is know as the Ornstein-Uhlenbeck process. It defines an elastic random walk around

a trend, with a mean reverting characteristic. Given the set of information $\mathcal{F}_s$ at time s, the short rate $r(t)$ is distributed as

$$r_t|\mathcal{F}_s \sim N\Big(r_s e^{-a(t-s)} + b\big(1-e^{-a(t-s)}\big), \frac{\sigma^2}{2a}\big(1-e^{-2a(t-s)}\big)\Big) \tag{2}$$

The zero cupon bond price in time t with maturity in T can be explicitly computed concluding that the Vasicek Model is an affine model whose the solution is

$$P(t,T) = A(t,T)e^{-B(t,T)r_t}; \tag{3}$$

where

$$A(t,T) = \exp\Big(\big(b-\frac{\sigma^2}{2a^2}\big)(B(t,T)-T+t) - \frac{\sigma^2}{4a}B(t,T)^2\Big),$$
$$B(t,T) = \frac{1}{a}\Big(1-e^{-a(T-t)}\Big).$$

Also, from (3) we obtain the continuously compounded spot interest rate by

$$R(t,T) := -\frac{\log P(t,T)}{T-t} = \hat{A}(t,T) + \hat{B}(t,T)r_t, \tag{4}$$

where $\hat{A}(t,T) = -\frac{\log A(t,T)}{T-t}$ and $\hat{B}(t,T) = \frac{B(t,T)}{T-t}$.

Concerning option pricing in this model, Jamshidian [15] derives a closed-form solution for European call options with strike K and maturity S for a zero coupon bond with maturity T, by the formula

$$C(t,S,K,T) = P(t,T)N(h) - KP(t,S)N(h-\sigma_p).$$

Here

$$h = \frac{1}{\sigma_p}\log\Big(\frac{P(t,T)}{KP(t,S)}\Big) + \frac{\sigma_p}{2}; \quad \sigma_p = \frac{\sigma}{a}(1-e^{-a(T-S)})\sqrt{\frac{1-e^{-2a(S-t)}}{2a}}.$$

2.2. **Calibration in the Vasicek model.** The basic problem is to estimate the short rate given access only to daily yield bonds, to subsequently estimate the parameter set. With the purpose of obtaining the information included in the interest rates, it is necessary to use n different maturities in the term structure. As this would give rise to an identification problem, we allow differences between observed rates and the model rates, introducing error variables, as suggested in [6]. From equations (2)-(4), and assuming that the yields are observed at discrete time points, we obtain the following model, that can that be represented in state-space form:

$$r_{t_i} = b\big(1-e^{-a\Delta t}\big) + e^{-a\Delta t}r_{t_i} + \epsilon_{t_i}, \tag{5}$$
$$R_{t_i} = \hat{A}_{t_i} + \hat{B}_{t_i}r(t_i) + \nu_{t_i}, \tag{6}$$

where $R_{t_i} \in \mathbb{R}^n$, $\hat{A}_{t_i} \in \mathbb{R}^n$, $\hat{B}_{t_i} \in \mathbb{R}^n$, $\nu_{t_i} \in \mathbb{R}^n$; the noise process is $\epsilon_{t_i}|\mathcal{F}_{t_{i-1}} \sim N(0,Q)$ with $Q = \frac{\sigma^2(1-e^{-2a\Delta t})}{2a}$; and the measurement noise is $\nu_{t_i} \sim N(0,S)$, with

$$S = \begin{pmatrix} s_1^2 & 0 & \cdots & 0 \\ 0 & s_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & s_n^2 \end{pmatrix}.$$

The Kalman filter provides the optimal estimator for a large class of problems and a very effective and useful estimator in our case. The filter is essentially a set

of equations that implement a predictor-corrector type estimator of the random variable r_{t_i}. The initial prediction $r_{t_i|t_{i-1}}$ is obtained using the distribution of the short rate, conditional on the previous estimated values and this estimate is updated to $r_{t_i|t_i}$ using the information provided by the observed yield vector.

More precisely, is composed by two *prediction* equations:

$$\begin{aligned} r_{t_i|t_{i-1}} =& b\big(1-e^{-a\Delta t}\big)+e^{-a\Delta t}r_{t_i|t_{i-1}}, \\ P_{t_i|t_{i-1}} =& e^{-a\Delta t}P_{t-1|t-1}e^{-a\Delta t}+Q, \end{aligned}$$

where $r_{t_i|t_{i-1}} = \mathbf{E}(r_{t_i}|\mathcal{F}_{t_{i-1}})$ and $P_{t_i|t_{i-1}} = \mathbf{E}((r_{t_i} - r_{t_i|t_{i-1}})(r_{t_i} - r_{t_i|t_{i-1}})|\mathcal{F}_{t_{i-1}})$, and three *update* equations:

$$\begin{aligned} K_{t_i} &= P_{t_i|t_{i-1}}\hat{B}_{t_i}^T(\hat{B}_{t_i}P_{t_i|t_{i-1}}\hat{B}_{t_i}^T+R)^{-1}, \\ r_{t_i|t_i} &= r_{t_i|t_{i-1}}+K_t(R_t-\hat{A}_{t_i}-\hat{B}_{t_i}y_{t_i|t_{i-1}}), \\ P_{t_i|t_i} &= (Id-K_{t_i}\hat{B}_{t_i})P_{t_i|t_{i-1}}, \end{aligned}$$

where K_{t_i} is the gain of the Kalman filter when using the rates given by the market. This set of equations should be iterated for each time t_i, being also necessary to give initial values in the prediction equations, that we denote by $y_{0|0}$ and $P_{0|0}$. With this new time series we construct the log-likelihood function to find the optimal parameter set

$$\log L(R,\Psi)=\sum_{t=1}^{T}\log(R_{t_i}|F_{t_{i-1}})=-\frac{nT}{2}\log(2\pi)-\frac{1}{2}\sum_{t=1}^{T}\log|H_{t_i}|-\frac{1}{2}\sum_{t=1}^{T}\zeta_t^T H_{t_i}^{-1}\zeta_{t_i} \tag{7}$$

where $\zeta_{t_i} = z_{t_i} - \hat{A}_{t_i} - \hat{B}_{t_i}y_{t_i|t_{i-1}}$ and $H_{t_i} = \hat{B}_{t_i}P_{t_i|t_{i-1}}\hat{B}_{t_i}^T + S$.

It is necessary to use nonlinear optimization techniques to find the maximum in (7). In this work, we choose the technique expectation maximization. There is an extensive literature related to this subject, stand out the first article on the topic written by Rudolf Kalman [16] and the book [10]. In particular, the Kalman filter is used in estimation of term structure models in [6] and [14].

2.3. **The Black-Derman-Toy model (BDT).** The Black-Derman-Toy one-factor model [3] is one of the most used yield-based models to price bonds and interest-rate options. The model is arbitrage-free and was developed to match the observed term structure of yields on zero coupon bonds and their corresponding volatilities. There also exists a continuous time version of the BDT model where the short-rate process has the form

$$r(t) = U(t)e^{\sigma(t)W(t)} \tag{8}$$

where $U(t)$ is the median of the short-rate distribution at time t, $\sigma(t)$ the short-rate volatility, and $\{W(t)\}$ a standard Brownian motion as before. The dynamics of the logarithm of the short rate is given by a stochastic differential equation (see [4]). Unfortunately, due to the log-normality of $r(t)$ in (8), analytic solutions are not available for bond option prices and numerical procedures like binomial tree are required.

2.4. **Calibration of the BDT model.** We can use a binomial tree to calibrate the model parameters to fit both the current term structure of yields on zero coupon bonds, and the volatility structure. In this model, the short-term rate for the next period can have only one of two possible values, r_u or r_d where $r_u > r_d$. To reduce

the number of nodes a restriction is imposed which forces an up move followed by a down move to result in the same future interest rate as a down move followed by an up move. This makes the binomial tree computationally tractable since the number of nodes at each time step increases by only one node. An up move in the short-term interest rate has a probability q, so the corresponding down move has a probability of $1-q$. We use $q = 0.5$. When refering to nodes in a binomial tree, we use the index notation in Figure 2.

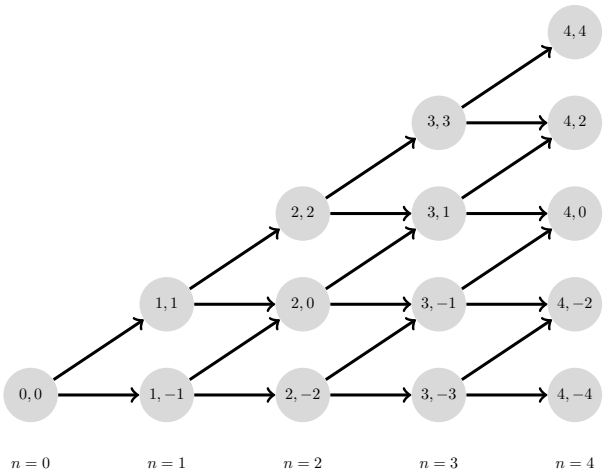


FIGURE 2. Example of a four period binomial tree.

Let the unit time be divided into M periods of length $\Delta t = 1/M$ each. At each period n, corresponding to time $t = n/M = n\Delta t$, there are $n+1$ states. These states range according to $j = -n, -n+2, \ldots, n-2, n$. At the present period $n = 0$, there is a single state $i = 0$. Let $r(n, j)$ denote the annualized one-period rate at period n and state j. Define the variable $X_k = \sum_{j=1}^{j=k} y_j$ where $y_j = 1$ if an up move occurs at period k and $y_j = -1$ if a down move occurs at period k. The variable X_k gives the state of the short rate at period k. At any period k, the X_k has a binomial distribution with mean zero and variance k. It follows that $X_k\sqrt{\Delta t}$ has the same mean and variance as the Brownian motion $W(t)$. X_k has independent increments, the binomial process $X_k\sqrt{\Delta t}$ converges to the Brownian motion $W(t)$ as Δt approaches zero. The state of the short rate was denoted by j. Replacing X_k by j will lead to having $W(t)$ approximated as $j\sqrt{\Delta t}$. Now, replacing t by n (with $n = t/\Delta t$) gives the discrete version of $r(t)$

$$r_{n,j} = U(n)e^{\sigma(n)j\sqrt{\Delta t}} \text{ with } j = -n, -n+2, ..., n-2, n.$$

To calibrate the tree to the term structure and the volatility we use the Arrow-Debreu prices where it represents prices of primitive securities and forward induction. Let $G(n, i, m, j)$ denote the price at period n and state i of a security that has a cash flow of unity at period m ($m \geq n$) and state j. Note that $G(m, j, m, j) = 1$ and that $G(m, i, m, j) = 0$ for $i \neq j$. We can compute $G(n, i, m+1, j)$ with the

forward induction:

$$\begin{cases} \frac{1}{2}p(m,j-1)G(n,i,m,j-1) & j = m+1. \\ \frac{1}{2}[p(m,j+1)G(n,i,m,j+1) + p(m,j-1)G(n,i,m,j-1)] & \mid j \mid < m-1. \\ \frac{1}{2}p(m,j+1)G(n,i,m,j+1) & j = -m-1. \end{cases}$$

where $p(m,j)$ is the one-period discount factor:

$$p(m,j) = e^{r_{m,j}\Delta t}$$

By intuitive reasoning, the forward induction function above states how we discount a cash flow of unity for receiving it one period later. Arrow-Debreu prices are the building blocks of all securities. The price of a zero-coupon bond which matures at period $m+1$ can be expressed in terms of the Arrow-Debreu prices and the discount factors in period m.

$$P(0,m+1) = \sum_j G(0,0,m,j)p(m,j)$$

The term structure $P(0,m)$, which represent the price today of a bond that pays unity at period m, can be obtained for all values of m, by the maximum smoothness criterion.

Once the binomial tree is calibrated, it is easy to calculate bond prices and bond option prices using backward induction. For example, assume that we want to price of a 4-period zero coupon bond in the model described in Figure 2. We known that at maturity the bond price is 100, so the nodes $(4,4)(4,2)(4,0),(4,-2)$ and $(4,-4)$ are equal to 100. To find the price at $(3,3)$ we apply the risk-neutral principle, that gives

$$G(0,0,3,3) = \frac{p(3,3)}{2}\left(G(0,0,4,4) + G(0,0,4,2)\right),$$

(i.e. the discounted average of the next two posible nodes). For all other nodes we act in the same way. The price at the node $(0,0)$ is the price of the zero coupon bond today. To calculate the price of an option we proceed in the same way, with the corresponding payoff instead of the bond price G.

3. **Empirical results.**

3.1. **Data.**

To model the USD and UI Uruguayan debt structure we use the respective daily yield curves in USD and UI released by BEVSA for dates between 7/1/2015 and 6/30/2016, with maturities up to 5 years. In the case of the Vasicek model, this historical data base is the input for the Kalman filter, to estimate the parameters of the model. In the BDT model, this historical series is necessary to estimate the volatility curve $\sigma_R(t)$ for $t \in [0,T]$ corresponding to the 6/30/2016.

3.2. **Option prices.**

We compute European call and put options prices for different strikes and maturities of $3, 6$ and 12 months, for zero coupon bonds issued in USD and UI, paying 100 at maturity[1]. In the USD case, our underling is a zero coupon bond with expiration on the 5/23/2019, that has a price of 94.17 (with a corresponding yield of 2.20%). In the UI case, our underling is a zero coupon bond with expiration on the 9/14/2018, that has a price of 89.23 (with a corresponding

[1]Put option prices, that can be obtained by parity arguments, are presented for comparison purposes.

yield of 5.16%). In both cases the strikes correspond to ±1% and ±2% w.r.t. the initial yield.

In Table 1 we present the computed prices. The parameters corresponding to the Vasicek model in (1) are $(a, b, \sigma) = (0.2289, 0.0713, 0.0104)$ for USD and $(a, b, \sigma) = (0.5782, 0.0523, 0.0121)$ for UI.

USD 3 months option prices

Strike	CALL		PUT	
	VK	BDT	VK	BDT
99.50	0.000	0.000	5.268	5.069
97.06	0.000	0.000	2.831	2.641
94.69	0.001	0.187	0.464	0.459
92.37	1.853	2.044	0.000	0.007
90.11	4.111	4.291	0.000	0.000

USD 6 months option prices

Strike	CALL		PUT	
	VK	BDT	VK	BDT
99.55	0.000	0.000	5.117	4.852
97.36	0.000	0.000	2.934	2.673
95.22	0.015	0.160	0.801	0.703
93.12	1.129	1.599	0.000	0.058
91.07	3.336	3.582	0.000	0.003

USD 1 year option prices

Strike	CALL		PUT	
	VK	BDT	VK	BDT
99.65	0.000	0.000	4.561	4.224
97.94	0.000	0.000	2.867	2.538
96.26	0.000	0.069	1.204	0.950
94.61	0.441	0.918	0.011	0.170
92.99	2.034	2.373	0.000	0.023

UI 3 months option prices

Strike	CALL		PUT	
	VK	BDT	VK	BDT
94.00	0.000	0.000	3.617	3.580
92.18	0.000	0.000	1.817	1.782
90.40	0.117	0.240	0.173	0.259
88.65	1.675	1.711	0.000	0.001
86.93	3.377	3.405	0.000	0.000

UI 6 months option prices

Strike	CALL		PUT	
	VK	BDT	VK	BDT
94.76	0.000	0.000	3.341	3.128
93.15	0.000	0.002	1.766	1.569
91.58	0.086	0.267	0.316	0.300
90.03	1.286	1.484	0.001	0.008
88.51	2.773	2.958	0.000	0.000

UI 1 year option prices

Strike	CALL		PUT	
	VK	BDT	VK	BDT
96.25	0.000	0.000	2.673	2.229
95.10	0.000	0.005	1.573	1.136
93.95	0.037	0.231	0.511	0.277
92.83	0.619	1.047	0.002	0.021
91.71	1.669	2.086	0.000	0.001

TABLE 1. Option prices for zero coupon USD (left) and UI bonds (right). VK and BDT stand for Vasicek and Black-Derman-Toy models respectively

We observe that, in general, call options prices are higher for the BDT model than for Vasicek model, and respective put options are lower. This means that the BDT valuation density is slightly shifted to the right w.r.t. the Vasicek density, fact that can be checked computing futures prices, but the volatility of both models is approximately the same. We believe that the consideration of parametric models with more than one factor (see the model discussion in [18]) would provide prices closer to the BDT model, due to the improvement of the initial curve fitting.

4. **Conclusions.** In this work we obtain prices of European bond options, for USD nominated and inflation-indexed bonds in an emerging financial market. As the options market in not developed, we compare two different pricing methodologies: first we calibrate the Vasicek model (using a Kalman filter) and apply the Jamshidian option pricing formula; and second we price the same instruments with the help of the Black-Derman-Toy (BDT) tree model. Bond prices for calibration of both models are the same, and taken from the Electronic Stock Exchange of Uruguay (BEVSA). Both methodologies give relatively similar results, the Vasicek proposal being more robust and parsimonious, and the BDT being more flexible and arbitrage-free, seems to proviede more accurate prices. The work includes a thorough revision of the emerging markets situation in Latin America. The methodology proposed can be used to price European options in similar emerging markets, mainly as Brazil, México and Chile where the financial market are relatively more developed, and also to price exotic options that can be more attractive for risk hedging, such as American or Asian options.

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