Exchange Credit Risk: Measurement and Implications on the stability of partially dollarized financial systems

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Abstract

Some emergent economies present a high financial dollarization in loans and deposits, generating a specific risk in the banking activity. We quantify this exchange credit risk as the price of an option equivalent to this loan, and discuss the financial stability implications due to the (implicit) issuance of these options. The exchange rate is modeled through a Lévy process. The depth of the market depends on the type of the currencies involved. Whenever possible, we depart from option prices to calibrate a model, like in the EUR/USD market. But if the market is not liquid, as the USD/UYU market, we provide alternative pricing methodologies.

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1 Introduction

As certain emergent economies are highly dollarized a new type of financial risk appears. The dollarization of the economy has the following consequence in banking activity: as banks receive a high proportion of their deposits in foreign currency they adopt a strategy of giving loans also in foreign currency. But local agents receiving these credits have their income mainly in local currency. This moves the mismatch from the bank to the local agents, generating a type of risk we name in this paper: the *exchange credit risk*, known also as *currency-induced credit risk*. The exchange credit risk is defined as the expected loss derived from the fact of lending in foreign currency to agents whose income is mainly in local currency. The relevance of studying and having precise estimates of this type of risk allows the understanding of the vulnerability of high dollarized economies in the event of a devaluation. In case of a devaluation, a negative income effect is generated by the increase in the real value of debtor's obligations with respect to the value of their assets.

In (Pena 2009), Pena proposes a way of estimating the exchange credit risk by using Merton's Theorem through a concrete financial instrument. In order to valuate this instrument, the characteristics of the considered country should also be taken into account. Merton's Model (Merton 1974) is based on the non existence of currency mismatch, but it can easily be generalized to the case where the debtor gets a foreign currency loan. The credit risk is associated to the evolution of the exchange rate.

In order to model the exchange credit risk we assume that the borrower's asset value, denoted by A, is constant throughout the considered time period, without intermediate payments by the debtor. The market value of the borrower's equity at time T is $C_T = \max(A - S_T K^*, 0)$, where S_t is the value of the exchange rate and K^* is the value of the foreign currency debt. The market value of the debt in local currency at the end of the contract is expressed by

$$\begin{aligned} B_T = A - C_T &= A - \max(A - S_T K^*, 0) = \min(A, S_T K^*) \\ &= S_T K^* - \max(S_T K^* - A, 0) = S_T K^* - K^* \max\left(S_T - \frac{A}{K^*}, 0\right). \end{aligned} \quad (1)$$

Therefore, we conclude that the loan exposed to currency mismatch is equivalent to a long position in a risk-free loan and a short position in a call option where the strike is given by the relationship between the asset's value and the agreed value of the debt in dollars. If the exchange rate exceeds that value, the option would be executed by the borrower and the lender would obtain only the value of the debtor's asset.

In some Latin-American economies that present high financial dollarization Central Banks have focused on the study of credit exchange risk. Their aim is to evaluate the implications of this risk in the bank's and system solvency. This topic is included in papers (Azabache 2005) and (Pena 2009).

In references to works in dollarized economies, (De Nicolo et al. 2005) discuss different aspects of dollarization of the economy, in particular analyzing bank deposits and its consequences; (Holland et al. 2012) state that "dollarization is a rational response to the future inflation associated with investors expectations of default observed in highly indebted economies", providing empirical support with the help of panel data analysis. In (Carranza and Gomez 2009) a similar point of view is assumed, providing more empirical evidence; while (Honig 2009) discusses if the exchange rate regime has an important impact on dollarization and conclude that improved government quality reduce the dollarization while the exchange rate regime plays no direct role. Other related works include (Bartrman and Bodnar 2012), (Mihaljek and Packer 2010), and

(Winkelried and Castillo 2010).

Several variables play an important role in the determination of the exchange rate. Some of them are local interest rate, foreign interest rate, inflation, trade barriers, country productivity, real exchange rate equilibrium and money supply. In addition, the exchange rate is influenced by foreign currency flows; see (Mishkin 2004). These variables have the characteristic of being very dependent on its future expectation and they change with new information in the market. Hence, they produce large fluctuations in the exchange rate in short time periods, causing heavy tails and positive kurtosis in the empirical distribution.

Besides, in highly dollarized economies we observe great asymmetries between upward and downward movements of the exchange rate, which produces skewness in the empirical distribution. (Bussière and Tovar 2012) explore abrupt currency exchange moves and devaluation events in dollarized economies, motivating the introduction of processes with discontinuous trajectories.

Lévy processes and other stochastic processes with jumps have become popular for modeling market fluctuations because they reproduce the observed empirical properties of asset returns, providing better approximations than the classical Black-Scholes Model (Black and Scholes 1973). A lot of research papers related to Lévy Processes have been published to this date in various financial and applied mathematics journals, leading to a considerable literature in these issues. The mathematical prerequisites of the present paper can be found mainly in (Cont and Tankov 2003), (Kyprianou 2006) and (Mikosh 1998).

The rest of the paper is as follows. In section 2 we analyze how the exchange credit risk impacts on an asset value, on bank solvency and financial system stability in a dollarized economy. In section 3 we consider the mathematical model of the financial market based on Lévy processes. In subsection 3.1 we present the two models considered in the paper: Merton Model and Variance Gamma Model, in subsection 3.2 we discuss two methods in order to obtain the risk neutral measure: Esscher Transform and Calibration and in subsection 3.3 the Carr and Madan's method of option pricing through the Fast Fourier Transform. Empirical results corresponding to Euro/Dollar and Dollar/Uruguayan peso are the contents of section 4, and we conclude in section 5. Along the paper we adopt the ISO 4217 standard for denominations of currencies: USD for United States dollars, EUR for Euros, and UYU for Uruguayan pesos.

2 Implications for the stability of the financial system

The stability of a dollarized financial system can be influenced by fluctuations in the exchange rate by two basic channels: from each currency position banks have taken, and from possible variations in the value in USD of different assets denominated in this currency.

The following section explains how a dollar denominated asset experiments variations due to fluctuations in the exchange rate.

2.1 The value of a dollar denominated asset depends on the exchange rate

In the Introduction we established that a credit in USD given to an economic agent that has its income in the local currency can be replicated through a portfolio comprising a large position in a risk-free loan and a short position in a call option, where the strike is defined by the

rate between the value of the assets and the nominal value of the debt in USD. This result is given in equation (1). This relationship determines the value of the loan in local currency at its expiration time, in terms of market prices. If one considers an arbitrary time $t < T$, with $\tau = T - t$, the market value in local currency can be computed by

$$B_t = S_t K^* e^{-q\tau} - K^* \left(S_t e^{-q\tau} \Phi(d_1) - \frac{A}{K^*} e^{-r\tau} \Phi(d_2) \right) \\ = S_t K^* e^{-q\tau} (1 - \Phi(d_1)) + A e^{-r\tau} \Phi(d_2).$$

In the previous equation we have used the Black-Scholes pricing formula, with the usual expressions for d_1, d_2. The intuitive idea we want to present here is that with an increase in the exchange rate, the option is more likely to be exercised, so the market value of the debt in USD is a decreasing function of the exchange rate. In analytical terms, the market value of the loan in USD can be expressed as

$$B_t^* = \frac{B_t}{S_t} = K^* e^{-q\tau} - \frac{K^*}{S_t} \left(S_t e^{-q\tau} \Phi(d_1) - \frac{A}{K^*} e^{-r\tau} \Phi(d_2) \right).$$

Differentiating the above expression with respect to S_t, we obtain

$$\frac{\partial B_t^*}{\partial S_t} = -\frac{A}{S_t^2} e^{-r\tau} \Phi(d_2) < 0. \tag{2}$$

Therefore, an increase of the exchange rate automatically brings down the market value of debt signed by debtors who are subject to exchange credit risk. This is the second channel by which the exchange rate changes ultimately affects the solvency of a bank, and may in certain circumstances, affect the stability of the system. As we work in a more general framework than the option pricing model of Black-Scholes we remark that the above result holds for Lévy markets (see Proposition 3.1).

Finally, we must say that even when banks offer loans in dollars to economic agents who receive most of their income in dollars (as for instance exporters), there is still a possibility of currency mismatch, in the case that most of the costs of these agents evolve according to local currency. In this case, the debtor eventually would fall into insolvency if the local currency appreciates significantly against the dollar, which would also cause problems to the lender bank. In these cases, it can be shown that this type of loan can be represented as a long position in a risk-free credit and a short position in a put option, with strike related to costs that evolve linked to local currency. In view of the fact that dollar loans given by banks in highly dollarized economies imply that banks take short positions in call and put options on the exchange rate, some countries have eliminated the possibility of borrowing in that currency, even for exporters. Such is the case of Brazil.

2.2 Bank's solvency and its dependence on exchange rate

In this subsection we study how variations in the exchange rate may affect the bank's solvency. For this purpose, we first consider the case of a bank with the following characteristics:

A^*: foreign currency assets; A: local currency assets;
L^*: foreign currency liabilities; L: local currency liabilities.

In this way we can express total assets, bank's equity and the ratio of equity to total assets as follows

$$\begin{aligned} T_A &= A^*S + A, \\ P &= (A^* - L^*)S + (A - L), \\ k &= \frac{(A^* - L^*)S + (A - L)}{A^*S + A}, \end{aligned} \tag{3}$$

where T_A is total assets, P is equity, and k denotes the equity assets ratio, or capital asset ratio.

First case: Dollar denominated assets do not depend on exchange rate variation. In this case, the objective of each bank or regulator is to limit the exchange credit risk minimizing the dependence of k on exchange rate fluctuations. The goal is to make k, which depends on S, be independent of S. Departing from (3), we obtain that

$$((A^* - L^*) - kA^*)S + ((A - L) - kA) = 0.$$

If we assume that k does not depend on S, we obtain

$$(A^* - L^*) - kA^* = 0, \quad (A - L) - kA = 0.$$

Alternative expressions of the previous results are

$$A^* = \frac{L^*}{1-k}, \quad A = \frac{L}{1-k}. \tag{4}$$

This implies that the capital asset ratio is the same for all the currencies in which the bank works. Another way of looking at this fact is that a bank which operates with two currencies can be separated into two "pure" banks, both with the same capital asset ratio. A first consequence of this fact is that the position in each of the two currencies should be positive (not zero or closed) so that exchange risk assessed in this form is zero. An increase in the volatility of the exchange rate should not cause any change in bank's behavior, as their goal is to keep k independent of exchange rate fluctuations.

Second Case: Dollar denominated assets may depend on the exchange rate variation. We expressly not consider loans in dollars to agents that receive their income in dollars, but still could have currency mismatch if most of their costs evolve according to local currency. However, comments of the case will be made, if necessary.

Instead, as we consider it more relevant, we thoroughly develop the problematic of loans denominated in dollars taken by customers who have their income in local currency, which causes the so-called exchange credit risk. This kind of loans imply that banks have a risk-free long position in a dollar denominated asset and a short position in a call option where the notional is the amount of the loan. In this case, foreign currency assets, considering the discount due to the position for the options sold, can be expressed as

$$A^* - \alpha A^* \frac{C}{S}, \tag{5}$$

where α represents the percentage of assets that have been granted in loans that have exchange credit risk, C is the value of the option in local currency, and S is the exchange rate. With this in view, condition (4) has now the following form

$$A^* \left(1 - \alpha \frac{C}{S}\right) = \frac{L^*}{1-k}. \tag{6}$$

We can approximate the value of the call option in the neighborhood of its current value as

$$C \approx C_0 + \Delta dS + \frac{1}{2}\Gamma(\Delta dS)^2 + V d\sigma + \rho dr + \Theta dt.$$

Then, replacing this expression in (6), we have

$$A^* \left(1 - \alpha \frac{1}{S}\left(C_0 + \Delta dS + \frac{1}{2}\Gamma(\Delta dS)^2 + V d\sigma + \rho dr + \Theta dt\right)\right) = \frac{L^*}{1-k}. \tag{7}$$

It is assumed that banks control the position in each currency in the short term and the proportion of mismatched loans given in the medium term. Liabilities, both in level and in composition by currency, conform a fact about which little can be done by the bank in the medium term. On the other hand, the bank has a target in relation to k, which may be the regulatory minimum or some higher value. From the condition (7), we can study the impact of exchange rate in bank's stability.

- A jump in the exchange rate would produce a sharp drop in the market value of mismatched credits, resulting in a failure to achieve the goal of a capital asset ratio k. In terms of equation (5), an increase in S raises the value of option C and the ratio C/S increases.
- The extent to which a jump in the exchange rate can affect the system's stability is given by parameter α. This can be mitigated in part, to the extent that the bank has short positions in put options originated by loans in dollars to exporters with a cost structure that evolves with local currency. The delta of the call and put options sold have opposite sign, which favors the bank.
- An increase in volatility also reduces the value of the credits with currency mismatches, by increasing the value of call and put options sold. To achieve the goal of constant k, the bank should convert local currency assets to dollar denominated assets, this can increase volatility and exchange rate level, producing a vicious circle. To the extent that many banks are affected, it can generate a risk to the system's stability.
- An increase in the local interest rate would produce an increase in the value of the call option, therefore, this would produce a fall in the market value of the credits with mismatch, caused by agents with revenues in local currency. This effect can be offset by the reduction of the issued put options.
- The gamma effect becomes relevant when the option is near to expiration. An increase in gamma would increase the value of the call option, that would produce a fall in the value of credits in dollars. Something similar can be established for put options issued. In this case there is no hedge, the gamma effect is negative in both cases, by reducing the market value of the two types of credits.
- With respect to the parameter Θ in (7), it is usual that this effect diminishes near to the maturity of the credit. This means that this effect becomes important for long credits and this applies to both types of credits with "mismatch".

2.3 System's stability and its dependence on exchange rate

In terms of the foregoing analysis, two transmission channels may affect the system's stability.

First transmission channel: Since 2004 there has been a systematic tendency to depreciate the dollar over other currencies, causing that many banks have opted to keep most of their position in local currency. As the system is dollarized, with an important percentage of credits and deposits in dollars, a jump in the exchange rate can cause serious problems to the system's stability.

Second transmission channel: A significant percentage of system assets are constituted by credits with currency mismatches. This implies that the system's ratio α is large. In turn, this effect would be greater when:

- the exchange rate volatility is higher, as it was seen when volatility drops, the value of the two types of credit with mismatch falls. This is a type of systemic risk;
- the maturity of credits with currency mismatch is longer;
- the exchange rate presents sudden fluctuations or jumps;
- the development of local currency derivatives markets is still incipient.

On this last point, there is a discussion about whether the development of a derivatives market decreases the currency risk at the aggregate level. This discussion focuses just on the effect of derivatives on exchange rate volatility and aggregate exposure of economic agents to currency risk. However, the empirical evidence goes, in general, in the opposite direction: (Jadresic and Selaive 2005) show that activity in the forward market in Chile has not been associated with higher volatility in the exchange rate. They also found evidence to supports the view that development of the foreign exchange derivatives market is valuable to reduce aggregate currency risk. Other related work about Chile is (Chan 2005).

Additionally, an aspect that arises is that, although the development of a market for hedging instruments may decrease the risk at the aggregate level, the way it produces this result may be unsatisfactory. This would happen if there were participants who systematically have more information about exchange rate movements or if there were agents who have enough market power in order to make significant changes in the exchange rate. In the case of too small or too intervened exchange market, it is advisable to improve the system's efficiency and competence working with hedging instruments based on averages, as for instance Asian options that will be discussed later. Finally, in derivatives markets, speculators can take virtually unlimited positions and reduce the effectiveness of the Central Bank's intervention in the exchange market. However, this issue could be considered in the regulations of these markets. In this respect, see (Saxena and Villar 2008).

2.4 Stability considerations in a dollarized environment

The following list enumerates some relevant aspects that should be taken into account looking forward to stability in a highly dollarized environment.

1. Evaluation of the difference between the position of zero exchange risk, derived in equation (4), and the position that banks hold.
2. Evaluation of the percentage of assets that banks have granted to borrowers with currency mismatches, with identification of possible nettings derived from the emission of call and put options.

3. Assessing at a regulatory level if capital requirements or constitution of special provisions for loans with currency mismatches are required.

4. Evaluation of loan terms with implicit options included.

5. Evaluation of the volatility of the exchange rate and to what extent the monetary policy tends or not to smooth exchange rate volatility.

6. Evaluation of the fiscal policy and how this policy operates as a real-anchor for the exchange rate in the long term.

7. Evaluation of the development of the derivatives market, which of these are available and their relative cost.

In relation to item 2, many dollarized countries consider that this type of risk is not tolerable, and they directly forbid loans in dollars to agents whose income is in local currency. Such is the case of Argentina. In other countries, like Brazil, credit in dollars are forbidden to any economic agent, even exporters. In the case of Uruguay, there are credits that produce currency mismatch, but there is a more restrictive regulation in the constitution of provisions (expected losses) and in capital requirements (unexpected losses). Finally, in Peru there is a special system of provisions for credits with exchange credit risk. At the same time, the Superintendency of Banks, Insurance and AFP (Pension Fund Administration) of Peru have announced a change in the regulation of this risk for 2013. In the context of items 3, 5 and 6, there is an evaluation of what today has been called macroprudential regulation, and its effects on the aggregate or systemic exchange credit risk. Finally, as part of this paper, special emphasis is made on the possible use of options to cover this risk and the consequent need for a proper option valuation theory in highly dollarized systems, so this issue would be considered in detail in the next section.

2.5 Hedging instruments

A forward is a financial instrument usually available, or that can be synthesized in the money markets. Forwards are one of the most used instruments in South American economies; in fact, many banks do provide forward rate arrangements as a service to their customers. A future is another financial instrument available in the foreign exchange market. For most purposes it is reasonable to assume that forward and futures prices coincide. In practice, there are a number of factors, not reflected in theoretical models, that may cause forward and futures prices to be different; however, for short term contracts, these differences are in most cases negligible. Therefore, our purpose is to compare two different exchange risk hedging strategies: the use of options in relation to the use of forward contracts.

In the literature, there is no consensus on a universally preferable strategy to hedge currency risk. In (Han and Yin 2011) the potential benefit of hedging the currency exposure risk by forwards and options via stochastic programming is investigated. The authors conclude that both are important in a competitive economy and the option portfolios are an appropriated tool for currency risk control.

In particular, from the 2007-2008 financial crisis until now, while interest rate reached minimum values in central countries, interest rates in emerging countries were at high levels, in order to address the inflationary consequences that often entails the movement of capital from central to emerging countries. In this framework, the differential interest rate is very high, consequently

the forward price is very high too. Therefore, these instruments are not used as much as the economic authorities would like.

Consider for example the case of a debtor who needs protection against a jump in the exchange rate. The annual risk-free local interest rate is 10% and the risk-free interest rate in dollars to 1 year is 1%, the annual volatility of the exchange rate is 10% and term is being considered is 1 year.

In order to compare the cost of hedging with forwards in relation with the cost of hedging with options, we assume that the debtor take a loan in foreign currency and can hedge the exchange risk with a forward contract or an option contract. So, one must consider the implicitly interest rate of the loan measured in local currency, which is the reference currency for the debtor.

We also assume for this comparison, that the strike of the option is equal to the forward price. The interest rate could be represented by this two expressions

$$r_F = \frac{F_t^T}{S_t}(1+q) - 1, \qquad r_{OP} = \frac{\min(S_T; F_t^T)}{S_t}\frac{(1+q)}{(1-P)} - 1;$$

where

- r_F is the interest rate in local currency achieved through forward or future hedge;
- r_{OP} is the interest rate in local currency achieved through option hedge;
- S_T is the value of the exchange rate at the expiration of the hedge contract;
- S_t is the value of the exchange rate when the debtor take the hedge contract;
- q is the interest rate of the loan in foreign currency;
- P is the cost of the option, measured in foreign currency;
- F_t^T is the price of the forward and also the strike of the option.

If we assume that $S_t = 21$, and $q = 3\%$, Figure 1 represents the alternative cost related with the two hedge contracts. So, there is a gain area for the options contract, that mainly depends on two factors: the exchange rate at maturity S_T and the option price P (this last price depends on the volatility charged by the seller of the protection).

Even, when the volatility charged by the seller of the protection is high, the cost of hedging with an option could be less than the cost of hedging with a forward, depending on the final value of the exchange rate. Note, that if the exchange rate exceeds the strike, the cost of hedging with an option is equal to the cost of hedging with a forward plus the cost of the option.

Other characteristics that would be in favor of development of a market for currency options are (i) they are instruments more flexible than forwards; (ii) transfer the risk of loss but not the possibilities of profit; (iii) the use of long call and short put options allows the debtor to define ranges of variation for the exchange rate and also to reduce the cost of hedge; (iv) they are more accessible than forwards, as they do not require to have a credit facility or an initial margin.

Furthermore, options are better suited to the needs of the debtor, because he can choose up to which point to endure a devaluation. In the case of a forward the obtained rate of devaluation is determined by the market, and is not necessarily adequate to the debtor needs. In short, options are instruments tailored to the clients needs, and are more accessible than

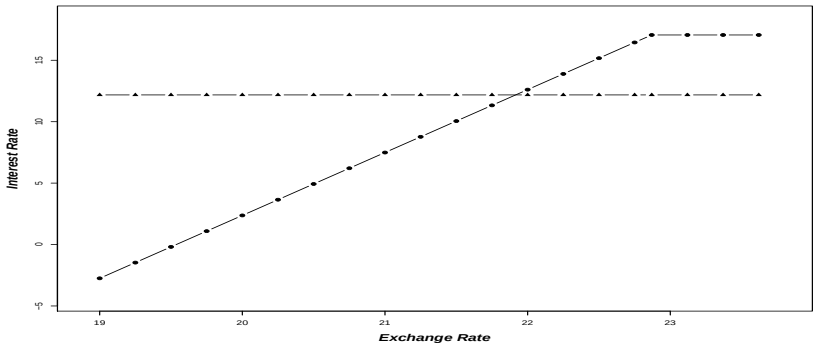


Figure 1: *Dots*: Interest rate in local currency achieved through option hedge. *Triangles*: Interest rate in local currency achieved through forward hedge.

forwards as they do not require credit facilities. Thus, options are particularly attractive for small and medium enterprises, and developing these instruments for highly dollarized economies is a pending issue.

Finally, when there is privileged information, or in small exchange markets where some agent can alter the level of the exchange rate, a way to improve the efficiency and acceptance of hedging instruments are Asian options on the exchange rate. They are cheaper than vanilla options because the volatility of a variable average is smaller than the volatility of the variable. The negative side of Asian options is that the debtor is not covered by the difference between the final exchange rate and the average exchange rate, remaining risk basis for the debtor. As forwards could be costly for long periods, it seems advisable to explore this possibility, rising the theoretical question of how to price these exotic options.

In conclusion, we propose to evaluate the possibility of covering the exchange credit risk using options, given that this financial instrument has some characteristics that make them preferable to forwards, the most used instruments in South American dollarized economies. Besides, options can be less expensive than forwards, depending on the value charged to the volatility and the final value of the exchange rate. This previous fact being enhanced when considering some kind of exotic options, as Asian options.

3 Financial market model

As we have discussed, and given the presence of devaluation events, which is a characteristic of emergent economies, we model the exchange rate by Lévy Processes. These processes include jumps in the time evolution of the asset, that can be interpreted as devaluation events.

3.1 Market model

We consider a market model with two possibilities of investment, a deterministic savings account, to model through a deterministic process $\left(B_t\right)_{0\leq t\leq T}$ that satisfies

$$B_t = B_0 e^{rt},$$

where $r \geq 0$ is the continuously compounded interest rate, and a random asset $\left(S_t\right)_{0\leq t\leq T}$, that satisfies

$$S_t = S_0 e^{X_t},$$

where $\left(X_t\right)_{0\leq t\leq T}$ is a Lévy process defined in a probability space $(\Omega, \mathcal{F}, \mathbf{P})$.

Here the Lévy process is characterized by its Lévy exponent

$$\psi(z) = \gamma z + \frac{1}{2}\sigma^2 z^2 + \int_{\mathbf{R}} (e^{zx} - 1 - zx\mathbf{1}_{\{|x|<1\}})\nu(dx), \tag{8}$$

or, equivalently, its triplet of characteristics (γ, σ, ν) that satisfy Lévy-Khinchine formula

$$\mathbf{E}(e^{zX_t}) = e^{t\psi(z)}.$$

For general reference on Lévy process we refer to books (Cont and Tankov 2003), (Kyprianou 2006), (Sato 1999) and (Shiryaev 1999). Below we present an auxiliary result.

Proposition 3.1. *Consider the value of a loan exposed to exchange credit risk defined by*

$$B_t^* = \frac{B_t}{S_t} = K^* e^{-q\tau} - \frac{K^*}{S_t}[\text{Call}]; \tag{9}$$

where the call option is written in a Lévy market model. Then, the same conclusion as in (2) *holds, i.e.*

$$\frac{\partial B_t^*}{\partial S_t} < 0.$$

Proof. According to (9) we have to verify

$$\frac{\partial\left([\text{Call}]/S_t\right)}{\partial S_t} > 0. \tag{10}$$

We have

$$\frac{\text{Call}_\text{t}}{S_t} = \frac{e^{-r(T-t)}}{S_t}\,\mathbf{E}\left[(S_T - K)^+ \mid S_t\right] = \frac{e^{-r(T-t)}}{S_t}\,\mathbf{E}\left[\left(S_t e^{X_{T-t}} - K\right)^+ \mid S_t\right]$$

$$= e^{-r(T-t)}\,\mathbf{E}\left[\left(e^{X_{T-t}} - \frac{K}{S_t}\right)^+ \mid S_t\right] = e^{-r(T-t)} G(S_t),$$

where

$$G(s) = \int_{\log(K/s)}^{\infty} \left(e^x - \frac{K}{s}\right) f_{X_{T-t}}(x)dx.$$

The fact that

$$G'(s) = \int_{\log(K/s)}^{\infty} \left(\frac{K}{s^2}\right) f_{X_{T-t}}(x)dx = \frac{K}{s^2}\,\mathbf{P}(se^{X_{T-t}} > K) > 0,$$

implies (10) concluding the proof. □

In the present paper we choose two particular models: the Merton's Jump Diffusion Model (Merton 1976) that has become a benchmark in jump modeling in finance, and the Variance Gamma Model introduced in (Madan et al. 1998), that has proved to adequately fit financial data in (Carr et al. 2002).

3.1.1 Merton's Jump Diffusion

In (Merton 1976) the evolution of an asset under the historical probability measure $\mathbf{P}$ is modeled as

$$S_t = S_0 \exp\big(\mu t + \sigma W_t + \sum_{i=1}^{N_t} \xi_i\big),$$

where $\big(W_t\big)_{0\leq t\leq T}$ is a Brownian motion, $\big(N_t\big)_{0\leq t\leq T}$ is a Poisson process of intensity $\lambda > 0$ and $\{\xi_i\}_{1\leq i\leq N}$ is a sequence of independent normal random variables with mean m and variance δ^2. The characteristic exponent (8) of Merton's Model is

$$\psi(z) = \mu z + \frac{\sigma^2 z^2}{2} + \lambda\big(e^{(\delta^2 z^2/2)+mz} - 1\big).$$

3.1.2 Variance Gamma

Introduced by (Madan et al. 1998), proposes to model the evolution of an asset under the historical probability measure $\mathbf{P}$ by

$$S_t = S_0 \exp\big(ct + m\gamma_t(1,k) + \sigma W_{\gamma_t(1,k)}\big),$$

where $\gamma_t(1,k)$ is a Gamma Process with mean rate 1 and variance rate k. Its characteristic exponent (8) of Variance Gamma is

$$\psi(z) = cz - \frac{1}{k}\log\big(1 - mkz - \frac{\sigma^2 k z^2}{2}\big).$$

3.2 From statistical to risk neutral probability

A characteristic of jump models is the incompleteness of the market, leading to the problem of the selection of the risk neutral probability pricing measure $\mathbf{Q}$. In this paper we present two ways of choosing the risk neutral probability measure.

3.2.1 Maximum Likelihood and Esscher Transform

A very popular way of choosing the risk neutral probability measure reported in the literature is the *Esscher Transform*, proposed by (Gerber and Shiu 1994). The historical probability measure $\mathbf{P}$ is estimated by maximum likelihood using an asset price time series. In order to find a probability measure $\mathbf{Q}$ such that the discounted asset price is a $\mathbf{Q}$-martingale, the Esscher Transform method defines

$$\frac{d\,\mathbf{Q}^{ET}}{d\,\mathbf{P}} = e^{\theta X_T - \psi(\theta)T}.$$

As $\mathbf{E}(e^{\theta X_T - \psi(\theta)t}) = 1$ and $\mathbf{P}(e^{\theta X_T - \psi(\theta)T} > 0) = 1$ the probability measure $\mathbf{Q}^{ET}$ is correctly defined and equivalent to $\mathbf{P}$. In order to obtain that the process $(e^{-rt}S_t)_{0\leq t\leq T}$ is a $\mathbf{Q}$ martingale, the condition

$$\psi(\theta + 1) - \psi(\theta) = r, \tag{11}$$

should be satisfied, and this condition, due to the convexity of the real function $\psi(\theta)$, determines a unique value of θ, and in consequence, a unique risk neutral measure $\mathbf{Q}^{ET}$.

Once the existence and uniqueness of the pricing measure is established, we recall that the triplet of characteristic of the discounted stock under the new probability measure $\mathbf{Q}^{ET}$ can be obtained in terms of the corresponding triplet under $\mathbf{P}$.

3.2.2 Calibration

In case of availability of option prices written on the underlying asset, it is natural to obtain $\mathbf{Q}$ directly from these prices, taking advantage of the implied information they contain. The method of choosing the risk neutral probability measure $\mathbf{Q}_{\theta^*}$ that reproduces the traded option prices is known as *calibration*.

Given an exponential Lévy model defined by its characteristic triplet $\big(\gamma(\theta), \sigma(\theta), \nu(\theta)\big)$, where θ represent the set of model parameters, observed prices C_i of call options for maturities T_i and strikes K_i, $i \in I$, the calibration consists in finding

$$\theta^* = \arg \min_{\mathbf{Q}_\theta \in Q} \sum_{i=1}^{N} \omega_i \left| C^\theta(K_i, T_i) - C_i \right|^2, \tag{12}$$

where C^θ denotes the call option price computed for the exponential Lévy model with triplet $\big(\gamma(\theta), \sigma(\theta), \nu(\theta)\big)$, the parameters ω_i are weights and Q is the set of martingale measures.

3.3 Pricing European options via Fourier Transform

In the Black-Scholes Model call and put option prices are explicitly given by the Black-Scholes formula, but in exponential Lévy models the probability density is generally not known in closed form. Therefore there is no explicit formula for call and put option prices. However, Lévy-Khinchine formula allows to use pricing techniques based on the Fourier transform. In this paper, we use the Carr and Madan's method, as presented in (Cont and Tankov 2003).

3.3.1 Carr and Madan's method

Let X_t be a Lévy process and $S_0 = 1$ (all prices are expressed in units of the underlying). We compute the price of an European call with strike K and maturity T in the exponential Lévy model. Denote $k = \log K$ the logarithm of the strike. To compute the call option price

$$C(k) = e^{-rT} E^{\mathbf{Q}}\Big(\big(e^{rT+X_T} - e^k\big)^+ \Big),$$

is necessary to express its Fourier Transform in terms of the characteristic function of X_T and to find the option prices for a range of strikes by Fourier inversion. The function $C(k)$ is not integrable but the idea is to compute the Fourier transform of the time value of the option

$$Z_T(k) = e^{-rT} E\Big(\big(e^{rT+X_T} - e^k\big)^+ \Big) - \big(1 - e^{k-rT}\big)^+ =$$

$$e^{-rT} \int_{-\infty}^{\infty} (e^{rT+x} - e^k)(1_{k \leqslant x+rT} - 1_{k \leqslant rT}) \rho_T(x) dx.$$

Applying the Fourier Transform, the option prices can be found by inverting the Fourier Transform

$$Z_T(k) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-ivk} \xi_T(v) dv,$$

where $\xi_T(v) = e^{ivrT} \frac{\Phi_T(v-i)-1}{(1+iv)iv}$. More details in (Cont 2001).

3.3.2 Numerical evaluation of the inverse Fourier Transform

In order to apply this method to empirical data, we need to approximate the inverse Fourier transform by its discrete Fourier transform. We have

$$\begin{aligned} Z_T(k) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-ivk} \xi_T(v) dv &\approx \frac{1}{2\pi} \int_{-A/2}^{A/2} e^{-ivk} \xi_T(v) dv \\ &\approx \frac{1}{2\pi} \frac{A}{N} \sum_{j=0}^{j=N-1} w_j \xi_T(v_j) e^{-iv_j k}, \end{aligned}$$

where $v_j = -A/2 + j\Delta$, $\Delta = A/N - 1$ is the discretization step and w_j are weights corresponding to the chosen integration rule. Setting $k_n = k_0 + \frac{2\pi n}{n\Delta}$, we see that the sum in the last term becomes a discrete Fourier transform

$$Z_t(k_n) \approx \frac{1}{2\pi} \frac{A}{N} e^{ik_n A/2} \sum_{j=0}^{j=N-1} w_j \xi_T(v_j) e^{-2\pi inj/N}, \tag{13}$$

that allows to use *Fast Fourier Transform* algorithm.

4 Empirical Results

As we have seen in the Introduction and following Merton, exchange credit risk can be quantified through option prices. As USD/UYU market is not enough developed and there are no currency options, we cannot benefit from the implicit information in option prices, useful in many studies. For this reason, in subsection 4.1, we consider EUR/USD option prices in order to calibrate the pricing probability measure $\mathbf{Q}_{\theta^*}$ corresponding to this market, with the purpose of comparing it with the measure $\mathbf{Q}^{ET}$ provided by the Esscher Transform. In subsection 4.2, we present the Uruguayan case estimating the exchange credit risk using the corresponding $\mathbf{Q}^{ET}$ measure. Comparing sections 4.1 and 4.2 we expect to have an idea of the precision of the Esscher Transform methodology used in option pricing.

4.1 The EUR/USD market

Introduced in January 1, 2002, the EUR has become a relevant currency, giving a new alternative for trade and investment in the world. We use option prices provided by a private bank. The interest rates correspond to United State interest rate and the Euribor (Euro Interbank Offered Rate).

4.1.1 Maximum Likelihood and Esscher Transform

In this subsection we estimate the Variance Gamma parameters of the model to compute option prices and we compare the results with market option prices. Data correspond to options with maturity on March 14, 2011.

To compute the parameters of the historical probability measure $\mathbf{P}$ we use maximum likelihood estimation, as implemented in the package `VarianceGamma` of *The R Project for Statistical Computing*.

The obtained results are

$$\boxed{c = 0.08186 \qquad m = -0.09699 \qquad \sigma = 0.12303 \qquad k = 0.11840.}$$

With these parameters we compute the risk neutral probability measure $\mathbf{Q}^{ET}$ by the Esscher Transform. The solution of the equation (11) is $\theta^* = 0.446397$, therefore the parameters of $\mathbf{Q}_{\theta^*}$ are

$$\boxed{c_{\theta^*} = 0.08186 \qquad m_{\theta^*} = -0.10430 \qquad \sigma_{\theta^*} = 0.12336 \qquad k_{\theta^*} = 0.11840.}$$

Finally, we apply the Carr and Madan's method (Cont and Tankov 2003) to obtain the theoretical option prices. In order to do this, we numerically evaluate the inverse Fourier Transform to compute the time value of the option $Z_T(k)$ by (13). The results are shown in Figure 2.

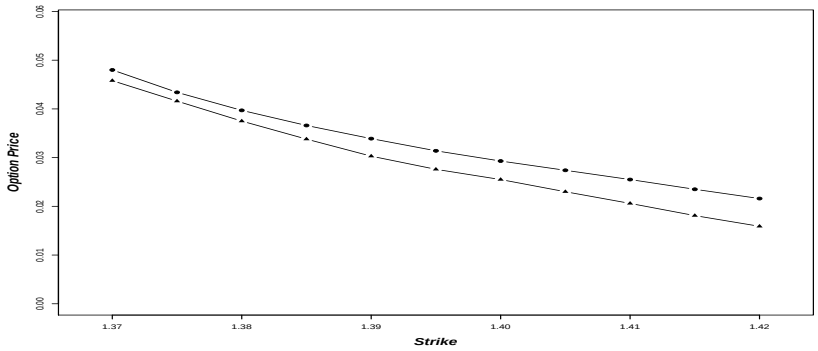


Figure 2: *Dots*: Option prices through Esscher Transform. *Triangles*: Market option prices.

In this case, the computed prices exceed the market prices for all strikes. The largest differences are for higher values of the strikes.

4.1.2 Calibration in Variance Gamma Model

In this subsection we calibrate the risk neutral probability measure $\mathbf{Q}_{\theta^*}$ of the Variance Gamma model by (12). The minimization is carried out with the help of the command `nls` of *The R Project for Statistical Computing*.

We use option market prices with maturity on March 14, 2011. The equivalence between the probability measures $\mathbf{P}$ and $\mathbf{Q}_{\theta^*}$ imply that the corresponding parameter c coincide. For this reason we set $c = 0.08186$.

The results are

$$\boxed{c = 0.08186 \qquad m = -0.6729 \qquad \sigma = 1.0852 \qquad k = 0.3363}$$

and are shown in Figure 3.

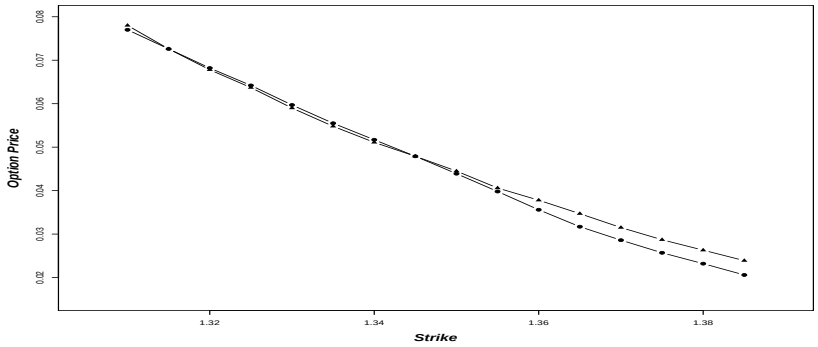


Figure 3: *Dots*: Option prices through calibration in the Variance Gamma model. *Triangle*: Market option prices.

4.1.3 Calibration in Merton's Jump Diffusion Model

In this subsection we calibrate the risk neutral probability measure $\mathbf{Q}_{\theta^*}$ corresponding to Merton's Jump Diffusion Model, following (12). In order to perform this calibration we use option market prices with maturity December 13, 2010.

The results are

$$\boxed{\mu = 0.0599 \quad \sigma = 0.0316 \quad \lambda = 1.2786 \quad m = -0.0489 \quad \delta = 0.3175}$$

and the comparison with market prices are shown in Figure 4. We conclude that calibration gives reasonable results, specially considering the strikes at the money.

4.2 The USD/UYU market

The Uruguayan banking system has certain peculiarities, the market participation of the four largest banks is very high and the state's participation exceeds 50%.

The economic crisis of 2002 in Uruguay produced an important decrease of bank deposits, mainly belonging to non-resident investors. In later years the ratio between deposits of residents

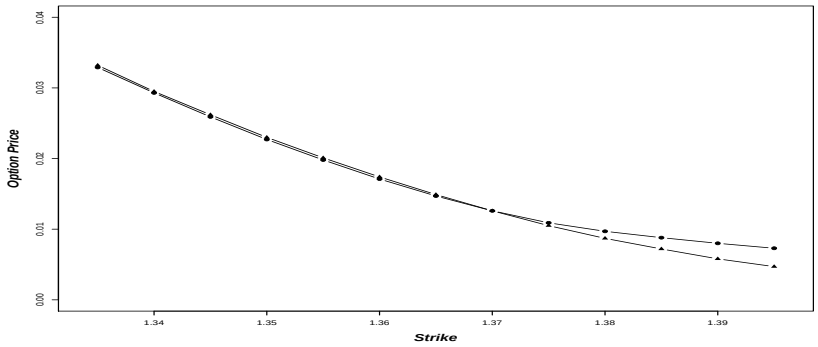


Figure 4: *Dots*: Option prices through calibration in the Merton's Jump Diffusion. *Triangle*: Market option prices.

and non-residents somehow stabilized. Furthermore, after this crises, the behavior of depositors changed, in particular, time deposits decreased and deposits in saving accounts increased.

A main feature of the Uruguayan banking system consists in its high financial dollarization, both in deposits and loans. Although in recent years the country managed, with the help of certain policies, to partially revert this phenomenon, at present dollarization remains around 70% in assets and liabilities. This feature has some evident drawbacks, because Uruguay is a small economy, with no significance in international markets, but nevertheless receives external shocks.

Based in the methodology proposed in the introduction to the Uruguayan case, we obtain that a loan in USD taken by an agent whose income is in UYU, can be expressed as a risk-free loan in UYU and a call option to buy USD, in which the beneficiary is the debtor.

As we mentioned, the fact that in Uruguay there is not a deep options market, and in consequence no option prices are available, it is necessary to implement an alternative method of option pricing. We then choose the Esscher Transform methodology, consisting first in the estimation of the historical probability $\mathbf{P}$ based on historical prices, and second, in the computation of the pricing measure $\mathbf{Q}^{ET}$ as the Esscher Transform of the probability $\mathbf{P}$.

Due to the fact that the exchange rate in Uruguay has suffered large upwards movements due to devaluations occurred during periods of economical crisis, we consider that jump models, as the Variance Gamma Model, give a better performance in option pricing than Black-Scholes Model for this market. The Variance Gamma Model adds two parameters that correspond to the skewness and the excess of kurtosis, that are usually present in the distributions of asset returns.

We first estimate the parameters of the Variance Gamma Model using the interbank USD prices from January 4, 2010 to October 31, 2011. The results are

$$c = -2.2713 \qquad m = 0.8745 \qquad \sigma = 0.1367 \qquad k = 0.1184.$$

This parameters define the historical probability measure $\mathbf{P}$ and using the corresponding equation (11) we obtain $\theta^* = -0.5292$. We then compute the parameters characterizing the Esscher Transform risk neutral pricing probability $\mathbf{Q}_{\theta^*}$

Strike	3 months	6 months	9 months	12 months	15 months	18 months	21 months	24 months
17.000	3.1743	3.5904	3.9614	4.3212	4.6821	5.0383	5.3992	5.7712
17.250	2.9587	3.3785	3.7540	4.1188	4.4848	4.8461	5.2121	5.5893
17.500	2.7401	3.1637	3.5439	3.9137	4.2850	4.6514	5.0226	5.4049
17.750	2.5186	2.9461	3.3310	3.7060	4.0826	4.4542	4.8306	5.2183
18.000	2.2941	2.7256	3.1155	3.4957	3.8776	4.0522	4.6362	5.0292
18.250	2.0666	2.5023	2.8970	3.2825	3.6699	3.8472	4.4392	4.8376
18.500	1.8361	2.2760	2.6758	3.0666	3.4595	3.6399	4.2396	4.6435
18.750	1.6025	2.0468	2.4517	2.8480	3.2463	3.4293	4.0375	4.4470
19.000	1.3657	1.8146	2.2246	2.6265	3.0305	3.2162	3.8327	4.2479
19.250	1.1259	1.5793	1.9946	2.4021	2.8118	3.0004	3.6253	4.0461
19.500	0.8828	1.3410	1.7617	2.1748	2.5903	2.7818	3.4152	3.8419
19.750	0.6365	1.0995	1.5257	1.9445	2.3659	2.5604	3.2024	3.6349
20.000	0.3870	0.8550	1.2866	1.7112	2.1386	2.3361	2.9868	3.4243
20.250	0.1623	0.6072	1.0444	1.4750	1.9084	2.0921	2.7685	3.2128
20.500	0.0143	0.3377	0.7996	1.2181	1.6580	1.8787	2.5309	2.9820
20.750		0.1101	0.5506	0.9933	1.4390	1.6456	2.3232	2.7799
21.000		0.0001	0.2988	0.7477	1.1996	1.4095	2.0962	2.5592
21.250			0.0732	0.4989	0.9573	1.1702	1.8663	2.3357
21.500				0.2469	0.7117	0.9279	1.6334	2.1092
21.750				0.0743	0.4629	0.6825	1.3975	1.8798
22.000					0.2110	0.4277	1.1586	1.6474
22.250					0.0298	0.1696	0.9106	1.4063
22.500						0.0271	0.6793	1.1619
22.750							0.4343	0.9142
23.000							0.2083	0.7234
23.250							0.0058	0.5124
23.500								0.2991
23.750								0.0834

Table 1: Option prices in UYU given by the Variance Gamma model

$$c_{\theta^*} = -2.2713 \qquad m_{\theta^*} = 0.9359 \qquad \sigma_{\theta^*} = 0.1406 \qquad k_{\theta^*} = 0.1184.$$

Once the pricing measure is obtained, we apply the Carr and Madan's method in (Cont and Tankov 2003) to compute theoretical option prices. These prices are shown in Table 1. In order to test the obtained results, we also compute Black-Scholes option prices with a volatility σ equal to the standard deviation of the risk neutral process.

In Figure 5 we see the option prices with maturity $T = 1$ (one year). In this case, we conclude that option prices given by the Variance Gamma Model are significantly higher than the ones given by the Black-Scholes formula.

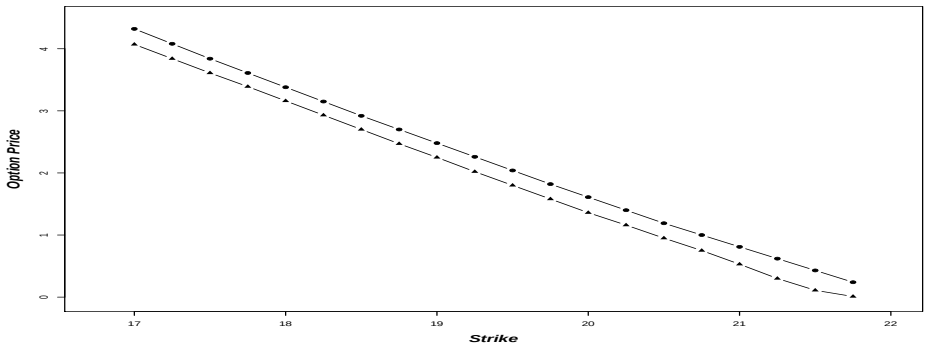


Figure 5: *Dots*: Option prices through Esscher Transform in the Variance Gamma model. *Triangle*: Option prices through Black-Scholes Model.

5 Discussion and Conclusion

We have two main motivations in order to analyze option pricing of currency exchange options.

- The first one is concrete, and comes from Merton's approach to a loan in foreign currency given to an agent whose income is in local currency. These calculations allow us to compute the values of the provisions for different debtors categories in the banking activity. Another possible application of the model is to quantify unexpected losses associated to loans using the Value at Risk methodology of the option. A third application can be to apply stress tests with the objective of estimating the potential losses under extreme conditions, for instance when a bank faces a sharp devaluation.
- Our second motivation is to analyze the impact of exchange rate fluctuations in asset value, bank's solvency, and global stability of a dollarized economy. This analysis suggests that the introduction of derivatives such as options can be a useful instrument to mitigate the exchange credit risk.

Some important aspects should be taken into account in orden to evaluate the stability in a highly dollarized environment (a) the position in each currency in relation to the position of zero exchange risk; (b) the percentage of assets that banks have granted to borrowers with currency mismatches; (c) the special provisions and capital requirements for loans with currency mismatches; (d) how the monetary policy tends to smooth the exchange rate volatility; (e) how the fiscal policy operates as a real anchor for the exchange rate; (f) the development of local currency derivatives market.

In relation to the last aspect, we propose to evaluate the possibility of developing an option's market to hedge the exchange risk. This is based on that the cost of hedging with an option, measured by the interest rate paid by the debtor, could be less than the cost of hedging with forward. Also, we can argue that the options are more flexible, transfer the risk of loss but not the possibilities of profit and not require to have a credit facility or an initial margin.

Accurate methods for option pricing can be useful for the development of this market. Due to the skewness and kurtosis reported in financial data, it is convenient to consider more flexible models than the classical Black-Scholes one, such as Lévy jump models. We then use Lévy financial models to compute European call option prices in foreign exchange option markets. Lévy processes provide both flexibility and computing possibilities. These financial market models are incomplete markets (as real markets), and have many pricing risk neutral probabilities. The required mathematical and computational tools to perform the option pricing include maximum likelihood estimation algorithms, Fast Fourier Transform to compute option prices and least-square minimization to calibrate Lévy models.

The liquidity of the markets depends on the currencies involved. For this reason we analyze option pricing in two different frameworks: the USD/UYU market where no options are available and the EUR/USD market, a typically liquid market, where different pricing methodologies are applied. The first one is based on the Esscher Transform and the historical probability estimation (for non-liquid markets, requires historical exchange rate data) and the second one is based on calibration (for liquid markets, requires option prices data).

The empirical results in the case EUR/USD show that the calibration approach gives better results than the Esscher Transform. This is reasonable due to the information provided by option prices. In the case of USD/UYU as the calibration approach is not applicable, we apply the Esscher methodology and compare the obtained results with the ones given by the classical

Black-Scholes model. Prices calculated in the Lévy model are larger than the ones given by the Black-Scholes formula. This can be a consequence of the presence of large movements in the exchange rate, usually upwards (i.e., devaluation events). These events are produced as Central Bank authority feels comfortable when buying USD with the propose of cushioning its fall, but when the exchange rate tends to rise it may present difficulties in selling the foreign currency to avoid the increase in its value. It is clear that other type of derivatives as Asian or American options deserve further treatment.

We expect that the proposed methodology can be a useful instrument in the management of exchange credit risk, providing a starting point for the development of a currency option market, and as an instrument towards stability in highly dollarized financial systems.

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