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RUIN PROBABILITIES FOR LÉVY PROCESSES WITH MIXED-EXPONENTIAL NEGATIVE JUMPS*

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Dedicated to the memory of José Luis Massera (1915–2002)

Abstract. We give a closed form of the ruin probability for Lévy processes, possible killed at a constant rate, with arbitrary, positive, and mixed exponentially negative jumps.

Key words. ruin probability, closed form, Lévy process, mixed-exponential distributions

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1. Introduction.

1.1. Let $X = \{X_t\}_{t\geqq 0}$ be a real-valued stochastic process defined on a stochastic basis $(\Omega, \mathcal{F}, \mathbf{F} = (\mathcal{F}_t)_{t\geqq 0}, \mathbf{P})$ that satisfies the usual conditions. Assume that X is càdlàg, adapted, $X_0 = 0$, and for $0 \leqq s < t$ the random variable $X_t - X_s$ is independent of the σ-field $\mathcal{F}_s$ with a distribution that depends only on the difference $t - s$. The stochastic process X is a process with stationary independent increments (PIIS) or a Lévy process. For $q \in \mathbf{R}$, $\psi(q)$ denotes the characteristic exponent of X given by the Lévy–Khinchin formula

$$\psi(q) = \frac{1}{t}\,\log \mathbf{E}e^{iqX_t} = ibq - \frac{1}{2}\,\sigma^2 q^2 + \int_{\mathbf{R}} (e^{iqy} - 1 - iqy\mathbf{1}_{\{|y|<1\}})\,\Pi(dy),$$

where b and $\sigma \geqq 0$ are real constants and Π is a positive measure on $\mathbf{R} - \{0\}$ such that $\int (1 \wedge y^2)\,\Pi(dy) < \infty$, called the Lévy measure. The function $\psi(q)$, $q \in \mathbf{R}$, completely determines the law of the process. For general reference on the subject see [4], [11], or [3].

1.2. Consider now a Lévy process with measure Π given by

$$\Pi(dy) = \begin{cases} \pi(dy) & \text{if} \quad y > 0, \\ \lambda \displaystyle\sum_{k=1}^{n} a_k \alpha_k e^{\alpha_k y}\, dy & \text{if} \quad y < 0, \end{cases} \tag{1}$$

where π is an arbitrary Lévy measure concentrated on $(0, \infty)$, $0 < \alpha_1 < \cdots < \alpha_n$, $a_k > 0$, for $k = 1, \ldots, n$ and $\sum_{k=1}^{n} a_k = 1$. The magnitude of the negative jumps of X is mixed exponentially distributed, with parameter α_k chosen with probability a_k, and they occur at Poissonian times with rate λ. As the process considered has a finite number of negative jumps on $[0, t]$, we consider a truncation function

$$h(y) = y\,\mathbf{1}_{\{0<y<1\}}. \tag{2}$$

Simple computations give

$$\psi(q) = iaq - \frac{1}{2}\sigma^2 q^2 + \int_0^{\infty} (e^{iqy} - 1 - iqh(y))\,\pi(dy) - \lambda \sum_{k=1}^{n} a_k\, \frac{iq}{\alpha_k + iq} \tag{3}$$

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with

$$a = b + \lambda \sum_{k=1}^{n} \frac{a_k}{\alpha_k} \left[1 - \frac{1+\alpha_k}{e^{\alpha_k}} \right].$$

We introduce now the Laplace exponent of X. For $p \in \mathbf{R}$, $p \leqq 0$, $p \neq -\alpha_k$, for $k = 1, \ldots, n$, denote

$$\kappa(p) = ap + \frac{1}{2}\,\sigma^2 p^2 + \int_0^\infty (e^{py} - 1 - ph(y))\,\pi(dy) - \lambda \sum_{k=1}^{n} a_k \frac{p}{\alpha_k + p}. \tag{4}$$

Considered as a function with complex domain, the characteristic exponent $iq \mapsto \psi(q)$ in (3) can be extended analytically to the complex strip $\{z = p + iq\colon p \in (-\alpha_1, 0]\}$ and, for $-\alpha_1 < p \leqq 0$, we have $\kappa(p) = \psi(p)$. As the Laplace transform of X_t when $-\alpha_1 < p \leqq 0$ is given by $\mathbf{E}e^{pX_t} = e^{t\kappa(p)}$, following [3] we call

$$\kappa(p) = t^{-1} \log \mathbf{E}e^{pX_t} \tag{5}$$

the Laplace exponent of X.

Let us examine the roots of the equation $\kappa(p) = 0$ for $p \leqq 0$. Since the integrand in (4) is convex in p for fixed y, we obtain that $\kappa(p)$ is convex on $(-\alpha_1, 0]$. From this it follows, when $\sigma > 0$, that under the condition

$$\kappa'(0-) = \lim_{p \to 0-} \frac{1}{p}\,\kappa(p) = a + \int_1^\infty y\,\pi(dy) - \lambda \sum_{k=1}^{n} \frac{a_k}{\alpha_k} > 0 \tag{6}$$

(where the integral can take the value ∞), the equation $\kappa(p) = 0$ has $n+1$ negative roots $-p_j$, $j = 1, \ldots, n+1$, that satisfy

$$0 < p_1 < \alpha_1 < p_2 < \cdots < p_n < \alpha_n < p_{n+1}. \tag{7}$$

Furthermore, observe that when $\gamma > 0$, equation $\kappa(p) = \gamma$ has always $n+1$ negative roots $-p_j$ satisfying (7).

Denote by $\tau(\gamma)$ an exponential random variable with parameter $\gamma > 0$, independent of X, and for $\gamma = 0$, put $\tau(0) = \infty$. We are interested in the random variables

$$M = \sup_{0 \leqq t < \tau(\gamma)} X_t \quad \text{and} \quad I = \inf_{0 \leqq t < \tau(\gamma)} X_t \tag{8}$$

called the supremum and infimum of the process, killed at rate γ if $\gamma > 0$ (see [3]).

The following result gives a closed formula for the ruin probability,

$$R(x) = \mathbf{P}\big\{\exists t \in [0, \tau(\gamma))\colon x + X_t \leqq 0\big\} = \mathbf{P}\{I \leqq -x\}, \qquad x \geqq 0, \tag{9}$$

for the process with characteristic exponent given by (3).

THEOREM 1.1. *Let X be a Lévy process with characteristic exponent given by* (3) *and $\sigma > 0$.*

(a) *Assume that condition* (6) *holds. Then $\mathbf{P}\{\lim_{t\to\infty} X_t = \infty\} = 1$, i.e., the process drifts to infinity.*

(b) *Let $\gamma \geqq 0$. Assume that condition* (6) *holds when $\gamma = 0$. Then the ruin probability* (9) *is given by*

$$R(x) = \sum_{j=1}^{n+1} A_j e^{-p_j x}, \qquad x \geqq 0, \tag{10}$$

where $-p_1, \ldots, -p_{n+1}$ are the negative roots of the equation $\kappa(p) = \gamma$ and the constants $A_1, \ldots, A_{n+1}$ are given by

$$A_j = \frac{\prod_{k=1}^{n}(1 - p_j/\alpha_k)}{\prod_{k=1,k\neq j}^{n+1}(1 - p_j/p_k)}, \qquad j = 1, \ldots, n+1. \tag{11}$$

Remark. As noticed by the referee (and can be confirmed from the proof), the existence of a real negative root in the interval $(-\infty, -\alpha_n)$ ensures the result (10) also in the case $\sigma = 0$.

1.3. Example. In the following example, the distributions of the supremum and infimum are obtained simultaneously, when jumps are completely specified. Let the process $X = \{X_t\}_{t \geqq 0}$ be given by

$$X_t = at + \sigma B_t + \sum_{k=1}^{N_t} Y_k, \tag{12}$$

where $B = \{B_t\}_{t \geqq 0}$ is a standard Brownian motion, $N = \{N_t\}_{t \geqq 0}$ is a Poisson process with parameter λ, $Y = \{Y_k\}_{k \in \mathbf{N}}$ is a sequence of independent and identically distributed random variables with density

$$d(y) = \begin{cases} \sum_{k=1}^{m} b_k \beta_k e^{-\beta_k y} & \text{if} \quad y > 0, \\ \sum_{k=1}^{n} a_k \alpha_k e^{\alpha_k y} & \text{if} \quad y < 0, \end{cases} \tag{13}$$

with $0 < \alpha_1 < \cdots < \alpha_n$, $0 < \beta_1 < \cdots < \beta_m$, a_k and b_k strictly positive for all k, and $\sum_{k=1}^{m} b_k + \sum_{k=1}^{n} a_k = 1$. The processes B, N, and Y are independent. The Laplace exponent (5) is given by

$$\kappa(p) = ap + \frac{1}{2}\sigma^2 p^2 + \lambda \sum_{k=1}^{m} b_k \frac{p}{\beta_k - p} - \lambda \sum_{k=1}^{n} a_k \frac{p}{\alpha_k + p}.$$

A simple application of Theorem 1.1 to X and $-X$ gives the following corollary.

COROLLARY 1.2. *Let X be given in* (12).

(a) *For $\gamma > 0$, the infimum I in* (8) *has a density given by*

$$f_I(x) = \sum_{k=1}^{n+1} A_k p_k e^{p_k x}, \qquad x \leqq 0, \tag{14}$$

where $-p_1, \ldots, -p_{n+1}$ are the negative roots of the equation $\kappa(p) = \gamma$, and $A_1, \ldots, A_{n+1}$ are given by (11).

(b) *For $\gamma > 0$, the supremum M in* (8) *has density given by*

$$f_M(x) = \sum_{k=1}^{m+1} B_k r_k e^{-r_k x}, \qquad x \geqq 0, \tag{15}$$

where $r_1, \ldots, r_{m+1}$ are the positive roots of the equation $\kappa(p) = \gamma$, and the coefficients are given by

$$B_j = \frac{\prod_{k=1}^{m}(1 - r_j/\beta_k)}{\prod_{k=1, k \neq j}^{n+1}(1 - r_j/r_k)}, \qquad j = 1, \ldots, m+1.$$

(c) *If $\kappa'(0) > 0$ (or $\kappa'(0) < 0$), then the process drifts to ∞ (or to $-\infty$), and the density of I (of M) in* (8) *with $\gamma = 0$ is given by* (14) (*or* (15)).

Remark. (a) and (b) in Corollary 1.2 can also be obtained from [10] by fractional expansion. In case of Theorem 1.1, in order to use this factorization method, complex variable results must be considered in order to count and locate the roots of $\psi(z) = \gamma$.

1.4. Results related to Theorem 1.1 and applications can be found in [1]. The distribution of the infimum of a process with no negative jumps was found by Zolotarev [13]; see also [8] or [3] and the references therein. Results giving a double Laplace transform of

finite time ruin can be found in [2], based on combinatorial methods and complex analysis. Rogozin [10] gives a factorization concerning the ruin of a killed process, extending combinatorial identities for random walks by Spitzer [12]. Theorem 1.1 generalizes previous work in the case of a Wiener process with negative exponential jumps [5] and of a Lévy process with negative exponential jumps [6]. Within the possible applications, the results are used to give closed solutions to optimal stopping problems for Lévy processes [7].

The method of proof is direct, along the lines of the barrier problem for simple random walks, consisting of finding a martingale with value 1 in the stopping region that vanishes if this region is not attained.

2. Proofs. First we introduce some notation and necessary facts. The infinitesimal generator of the process X with exponent in (3) is

$$(L^X f)(x) = a f'(x) + \frac{1}{2}\sigma^2 f''(x) + \int_0^\infty \big(f(x+y) - f(x) - f'(x)\,h(y)\big)\,\pi(dy)$$

$$+ \lambda \sum_{k=1}^{n} a_k \int_{-\infty}^{0} \big(f(x+y) - f(x)\big)\,\alpha_k e^{\alpha_k y}\,dy \tag{16}$$

with h in (2). The jump measure of X is

$$\mu^X = \mu^X(\omega, dt, dy) = \sum_s \mathbf{1}_{\{\Delta X_s \neq 0\}}\,\delta_{(s,\Delta X_s)}(dt, dy).$$

Its compensator is a deterministic measure given by $\nu = \nu(dt, dy) = dt\,\Pi(dy)$ with Π in (1). The process $M = \{M_t\}_{t \geqq 0}$ given by

$$M_t = \sigma B_t + \int_{[0,t]\times \mathbf{R}} h(y)(\mu^X - \nu),$$

where $B = \{B_t\}_{t \geqq 0}$ is a standard Brownian motion, is a martingale with no negative jumps, and X has the decompositions

$$X_t = M_t + at + \int_{[0,t]\times \mathbf{R}} (y - h(y))\,\mu^X \tag{17}$$

$$= M_t + at + \sum_{0 < s \leqq t} \Delta X_s \mathbf{1}_{\{\Delta X_s \geqq 1\}} - \sum_{k=1}^{N_t} Y_k, \tag{18}$$

where $N = \{N_t\}_{t \geqq 0}$ is a Poisson process with parameter λ, $Y = \{Y_k\}_{k \in \mathbf{N}}$ is a sequence of positive identically distributed random variables with density $d(y) = \sum_{i=1}^{n} a_k \alpha_k e^{-\alpha_k y}$, and B, N, and Y are independent.

The following result gives (20), the integrodifferential equation that the ruin probability must satisfy.

LEMMA 2.1. (a) *Let* $0 < p_1 < \alpha_1 < p_2 < \cdots < p_n < \alpha_n < p_{n+1}$, *and define the constants*

$$A_j = \frac{\prod_{k=1}^{n}(1 - p_j/\alpha_k)}{\prod_{k=1,k\neq j}^{n+1}(1 - p_j/p_k)}, \qquad j = 1, \ldots, n+1.$$

Then

$$\begin{cases} \displaystyle\sum_{j=1}^{n+1} A_j = 1, \\ \displaystyle\sum_{j=1}^{n+1} A_j \frac{p_j}{p_j - \alpha_k} = 0, \qquad k = 1, \ldots, n. \end{cases} \tag{19}$$

(b) *For the infinitesimal generator* L^X *in* (16), *the function* $R(x)$ *in* (10), *where now* $-p_1, \dots, -p_{n+1}$ *are the negative roots of the equation* $\kappa(p) = \gamma$ (*such that* (7) *holds*), *and* $x > 0$, *we have*

$$(L^X R)(x) = \gamma R(x) \tag{20}$$

with $R(x) = 1$ *if* $x < 0$.

Proof. (a) By the theorem on simple fractional expansion, there exist unique constants $A_1, \dots, A_{n+1}$ such that

$$\frac{\prod_{j=1}^{n}(1 + p/\alpha_j)}{\prod_{j=1}^{n+1}(1 + p/p_j)} = \sum_{j=1}^{n+1} A_j \frac{p_j}{p_j + p}. \tag{21}$$

Each equation of the linear system (19) is equation (21) evaluated at $p = 0$ and $p = -\alpha_k$ for $k = 1, \dots, n$, respectively. On the other hand, the relation for A_j $(j = 1, \dots, n+1)$ is obtained by standard methods, i.e., multiply (21) by $(p_j + p)/p_j$ and substitute $p = p_j$.

(b) Denote $f_p(x) = e^{-px^+}$, where $x^+ = \max(x, 0)$. As $\sum_{j=1}^{n+1} A_j = 1$, we can write

$$R(x) = \sum_{j=1}^{n+1} A_j f_{p_j}(x).$$

For $x > 0$,

$$\begin{aligned}(L^X f_p)(x) &= e^{-px}\left(\frac{1}{2}\,\sigma^2 p^2 - ap + \int_0^\infty (e^{-py} - 1 + ph(y))\,\pi(dy)\right) \\ &\quad + \lambda \sum_{k=1}^{n} a_k \int_{-\infty}^{0} (e^{-p(x+y)^+} - e^{-px})\,\alpha_k e^{\alpha_k y}\,dy \\ &= e^{-px}\left(\frac{1}{2}\,\sigma^2 p^2 - ap + \int_0^\infty (e^{-py} - 1 + ph(y))\,\pi(dy) + \lambda \sum_{k=1}^{n} a_k \frac{p}{\alpha_k - p}\right) \\ &\quad + \lambda \sum_{k=1}^{n} a_k e^{-\alpha_k x} \frac{p}{p - \alpha_k} = e^{-px}\kappa(-p) + \lambda \sum_{k=1}^{n} a_k e^{-\alpha_k x}\,\frac{p}{p - \alpha_k}.\end{aligned}$$

In conclusion, for $x > 0$,

$$\begin{aligned}L^X R(x) &= \sum_{k=1}^{n+1} A_j (L^X f_{p_j})(x) \\ &= \sum_{j=1}^{n+1} A_j e^{-p_j x}\kappa(-p_j) + \lambda \sum_{k=1}^{n} a_k e^{-\alpha_k x} \sum_{j=1}^{n+1} A_j\,\frac{p_j}{p_j - \alpha_k} = \gamma R(x),\end{aligned}$$

since $\kappa(-p_j) = \gamma$ for $j = 1, \dots, n+1$, and (19).

The next lemma summarizes the application of the Meyer–Itô formula to the process X with ψ in (3) and the function R in (10). Denote $Y = \{Y_t = x + X_t\}_{t \geqq 0}$.

LEMMA 2.2. *Let* $\tau_0 = \inf\{t \geqq 0\colon\ x + X_t \leqq 0\}$. *For* R *in* (10)

$$R(Y_{t\wedge\tau_0}) - R(x) = \int_0^{t\wedge\tau_0} (L^X R)(Y_{s-})\,ds + M(R)_{t\wedge\tau_0}, \tag{22}$$

with L^X *in* (16) *and* $M(R) = \{M(R)_t\}_{t\geqq 0}$ *given by*

$$M(R)_t = \int_0^t R'(Y_{s-})\,dM_s + \int_{[0,t]\times\mathbf{R}} W(s,y)(\mu^X - \nu), \tag{23}$$

is a local martingale, where $W(s,y) = R(Y_{s-}+y) - R(Y_{s-}) - R'(Y_{s-})\,h(y)$.

Proof. Since the function R is not $C^2(\mathbf{R})$ we will apply the Meyer–Itô formula [9, IV.51] denoting by $m(da)$ the signed measure that is the second derivative of R in the sense of distributions, when restricted to compacts:

$$R(Y_t) - R(x) = \int_0^t R'(Y_{s-})\,dY_s + \sum_{0<s\leqq t} \big(R(Y_s) - R(Y_{s-}) - R'(Y_{s-})\,\Delta Y_s\big) + \frac{1}{2}\int_{\mathbf{R}} l^Y(a,t)\,m(da), \tag{24}$$

where $l^Y(a,t)$ is the local time of the process Y at level a and time t.

First, observe that $m(da) = R''(a)\,da + R'(0+)\,\delta_0(da)$ with $R''(a)$ the second derivative of R if $a \neq 0$, $R'(0+)$ the right derivative of R at the point $x = 0$, and $\delta_0(da)$ the point mass at $x = 0$. Applying Corollary 1.2 to Theorem IV.51 in [9],

$$\int_{\mathbf{R}} l^Y(a,t)\,m(da) = \int_0^t R''(Y_{s-})\,d\langle Y^c, Y^c\rangle_s + R'(0+) l^Y(0,t). \tag{25}$$

In view of (17) and (25), as $d\,\langle Y^c, Y^c\rangle_s = \sigma^2 ds$, (24) gives

$$R(Y_t) - R(x) = \int_0^t R'(Y_{s-})\,dM_s + \int_0^t \left(\frac{1}{2}\,\sigma^2 R''(Y_{s-}) + aR'(Y_{s-})\right) ds + \frac{1}{2}\,R'(0+)\,l^Y(0,t) + \sum_{0<s\leqq t} W(s,\Delta X_s). \tag{26}$$

Now, we compensate the jumps in the last term. As R is bounded, the left-hand side in (26) is a special semimartingale. On the right-hand side, the first term is a local martingale and the second and third are continuous processes and, in consequence, locally integrable. We can apply II.1.28 in [4], obtaining

$$\sum_{0<s\leqq t} W(s,\Delta X_s) = \int_{[0,t]\times\mathbf{R}} W\mu^X = \int_{[0,t]\times\mathbf{R}} W(\mu^X - \nu) + \int_{[0,t]\times\mathbf{R}} W\nu = \int_{[0,t]\times\mathbf{R}} W(\mu^X - \nu) + \int_0^t \left[\int_{\mathbf{R}} W(s,y)\,\Pi(dy)\right] ds. \tag{27}$$

In order to evaluate (26) at τ_0, observe that $l^Y(0,\tau_0) = 0$ because $x + X_t > 0$ on the set $[0,\tau_0)$ and the local time is continuous in t (see [9, p. 165]). In view of (26) and (27), (22) follows.

Proof of Theorem 1.1. (a) In order to see that

$$\mathbf{P}\Big\{\lim_{t\to\infty} X_t = \infty\Big\} = 1 \tag{28}$$

consider the decomposition (18). Denote $d = \lambda \sum_{k=1}^n a_k/\alpha_k$. By (6) there exists $\varepsilon > 0$ such that

$$a + \int_1^\infty y\,\pi(dy) - (d+\varepsilon) > 0. \tag{29}$$

Consider $X^1 = \{X_t^1\}_{t\geqq 0}$ and $X^2 = \{X_t^2\}_{t\geqq 0}$ with

$$X_t^1 = at + M_t + \sum_{0<s\leqq t} \Delta X_s\,\mathbf{1}_{\{\Delta X_s \geqq 1\}} - (d+\varepsilon)\,t, \quad X_t^2 = (d+\varepsilon)\,t - \sum_{k=1}^{N_t} Y_k.$$

On the one hand, as X^1 has only positive jumps, by (29) and VII.1.2 in [3] we deduce $\mathbf{P}\{\lim_{t\to\infty} X_t^1 = \infty\} = 1$. On the other hand, as $\mathbf{E}(X_t^2) = \varepsilon t > 0$, we have $\mathbf{P}\{\lim_{t\to\infty} X_t^2 = \infty\} = 1$ and conclude (28).

Proof of (b). According to Lemma 2.2, $\{R(Y_{t\wedge\tau_0})\}$ is a semimartingale. So the quadratic covariation $[R(Y_{t\wedge\tau_0}), e^{-\gamma t}] = 0$ since the deterministic process $\{e^{-\gamma t}\}$ is continuous and has bounded variation. This gives

$$\begin{aligned} e^{-\gamma(t\wedge\tau_0)}R(Y_{t\wedge\tau_0}) - R(x) &= \int_0^{\tau_0\wedge t} e^{-\gamma s}\, dR(Y_{s\wedge\tau_0}) - \int_0^{\tau_0\wedge t} R(Y_{s\wedge\tau_0-})\,\gamma e^{-\gamma s}\, ds \\ &= \int_0^{\tau_0\wedge t} e^{-\gamma s}[L^X R(Y_{s-}) - \gamma R(Y_{s-})]\, ds + \int_0^{\tau_0\wedge t} e^{-\gamma s}\, dM(R)_s \\ &= \int_0^{\tau_0\wedge t} e^{-\gamma s}\, dM(R)_s \end{aligned}$$

with $M(R)$ in (23), by Lemma 2.1(b), as $Y_{s-} = x + X_{s-} > 0$ on $[0, \tau_0]$ and (20). Since the function R is bounded, taking expectations and limits as $t \to \infty$

$$R(x) = \mathbf{E}(e^{-\gamma\tau_0}R(x + X_{\tau_0})) = \mathbf{E}e^{-\gamma\tau_0} = \mathbf{P}\{\tau_0 < \tau(\gamma)\} = \mathbf{P}\{\exists t \in [0, \tau(\gamma))\colon x + X_t \leqq 0\}$$

concludes the proof.

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REFERENCES

[1] S. Asmussen, *Stationary distributions via first passage times*, in Advances in Queueing. Theory, Methods and Open Problems, J. H. Dshalalow, ed., CRC Press, Boca Raton, FL, 1995, pp. 79–102.

[2] G. Baxter and M. D. Donsker, *On the distribution of the supremum functional for processes with stationary independent increments*, Trans. Amer. Math. Soc., 85 (1952), pp. 73–87.

[3] J. Bertoin, *Lévy Processes*, Cambridge University Press, Cambridge, UK, 1996.

[4] J. Jacod and A. N. Shiryaev, *Limit Theorems for Stochastic Processes*, Springer, Berlin, Heidelberg, 1987.

[5] E. Mordecki, *Ruin probabilities and optimal stopping for a diffusion with jumps*, in Proceedings of the 4th Conference of Mathematics "Dr. Antonio A. R. Monteiro," E. L. Fernández Stacco et al. eds., Bahia Blanca, Argentina, 1997, pp. 39–48.

[6] E. Mordecki, *Optimal Stopping, Ruin Probabilities and Prophet Inequalities for Lévy Processes*, Prepublicaciones Matemáticas del Uruguay 2000/38, Centro de Matemática, Montevideo, 2000; also available online from http://premat.fing.edu.uy.

[7] E. Mordecki, *Optimal stopping and perpetual options for Lévy processes*, Finance Stoch., 6 (2002), pp. 473–493.

[8] N. U. Prabhu, *Stochastic Storage Processes*, Springer-Verlag, New York, 1980.

[9] P. Protter, *Stochastic Integration and Differential Equations*, Springer-Verlag, Berlin, Heidelberg, 1992.

[10] B. A. Rogozin, *On distributions of functionals related to boundary problems for processes with independent increments*, Theory Probab. Appl., 11 (1966), pp. 580–591.

[11] A. V. Skorokhod, *Random Processes with Independent Increments*, Kluwer Academic Publishers, Dordrecht, 1991.

[12] P. Spitzer, *A combinatorial lemma and its applications to probability theory*, Trans. Amer. Math. Soc., 82 (1956), pp. 323–339.

[13] V. M. Zolotarev, *The first passage time of a level and the behavior at infinity for a class of processes with independent increments*, Theory Probab. Appl., 9 (1964), pp. 653–662.

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