# TOPICS IN POLYNOMIAL DERIVATIONS AND THEIR AUTOMORPHISM GROUPS

IVÁN PAN AND ALVARO RITTATORE

Abstract. Let $\mathbb{k}$ be an algebraically closed field. It is well known that, if $n \geq 2$, one cannot endow the so-called *Affine Cremona Group* $\mathrm{ACr}(n)$, consisting of the automorphisms of the affine space $\mathbb{A}^n_{\mathbb{k}}$, with a structure of variety (or scheme), but $\mathrm{ACr}(n)$ can be seen as an ind-variety. Under this point of view, a lot of work has been done. For example, one may characterize when a closed subgroup $G \subset \mathrm{ACr}(n)$ inherits a structure of algebraic group — in this case we have a faithfull action of $G$ on the affine space. Analogously, one may ask for subgroup schemes of $\mathrm{ACr}(n)$.

On the other hand, by thinking of the elements in $\mathrm{ACr}(n)$ as $\mathbb{k}$-algebra automorphisms of $\mathbb{k}[x_1, \dots, x_n]$, we see that $\mathrm{ACr}(n)$ acts by conjugation on $\mathrm{Der}(\mathbb{k}[x_1, \dots, x_n])$, the vector space of the $\mathbb{k}$-derivations $D : \mathbb{k}[x_1, \dots, x_n] \to \mathbb{k}[x_1, \dots, x_n]$ — a $\mathbb{k}$-derivation is a linear map $D$ satisfying the Leibnitz rule $D(fg) = fD(g) + gD(f)$. The isotropy $\mathrm{Aut}(D)$ of such a $D$, which necessarily codifies important properties of its conjugation class, inherits a structure of closed ind-subvariety of $\mathrm{ACr}(n)$. It is natural to ask in which mesure (the equivalence class of) $D$ is determined by $\mathrm{Aut}(D)$ in the case where that subgroup acts on $\mathbb{A}^n_{\mathbb{k}}$ as an algebraic group (or group scheme).

This mini-couse focuses in introducing some tools to lead with this type of problem. More precisely: first we explain how to endow $\mathrm{Aut}(D)$ with a structure of closed ind-subgroup of $\mathrm{ACr}(n)$ and propose some problems relating to that. Subsequently, we describe the situation in dimension 2, where all is quite known. Finally, we give some results in higher dimension and propose some possible themes for studying.