A TWO-PLAYER ZERO-SUM PROBABILISTIC GAME THAT APPROXIMATES THE MEAN CURVATURE FLOW

Abstract: The aim of this talk is to introduce a two-players zero-sum probabilistic game whose value functions approximates the motion by mean curvature of a hypersurface that is the boundary of a connected and strictly convex domain, $S = \partial\Omega_0 \subset \mathbb{R}^N$, $N \geq 2$. We will use a level set approach to describe this geometric evolution. Assume that there is a real valued function, $u(x)$, defined for $x \in \Omega_0$, and consider the t superlevel sets of $u(x)$,

$$\Omega_t = \{x : u(x) > t\}, \quad t \geq 0.$$

Assume that $\partial\Omega_t$ is smooth, we have $\nabla u(x) \perp \partial\Omega_t$ and for a unitary vector $v \perp \nabla u(x)$ (notice that v is tangential to the hypersurface $\partial\Omega_t$) the quantity $-\langle D^2u(x)v, v\rangle$ gives the curvature of $\partial\Omega_t$ in the direction of v. Therefore, under these conditions, the mean of the principal curvatures of $\partial\Omega_t$ at a regular point is given by

$$\kappa = \sum_i \kappa_i = |\nabla u(x)| \mathrm{div}\Big(\frac{\nabla u}{|\nabla u|}\Big)(x) = \Delta u(x) - \Big\langle D^2u(x)\frac{\nabla u}{|\nabla u|}(x), \frac{\nabla u}{|\nabla u|}(x)\Big\rangle.$$

We consider the geometric evolution of the hypersurface $\partial\Omega_t$ moving its points in the direction of the normal vector (pointing inside the set Ω_t) with speed given by the mean curvature, $V = -\kappa$ on $\partial\Omega_t$. The elliptic equation associated is

$$\begin{cases} \Delta u(x) - \Big\langle D^2u(x)\frac{\nabla u}{|\nabla u|}(x), \frac{\nabla u}{|\nabla u|}(x)\Big\rangle = -1, & x \in \Omega_0, \\ u(x) = 0, & x \in \partial\Omega_0. \end{cases}$$

In collaboration with Irene Gonzálvez, Julio D. Rossi and Joger Ruiz Cases.

References

[1] R.V. Kohn and S. Serfaty. *A deterministic-control-based approach to motion by curvature.* Comm. Pure Appl. Math., 59(3), (2006), 344–407.

[2] L.C. Evans, J. Spruck, *Motion of level sets by mean curvature. I.* J. Differential Geom., 33(3) (1991), 635–681.

[3] I. Gonzálvez, A. Miranda, J. D. Rossi and J. Ruiz-Cases.*Finding the convex hull of a set using the flow by minimal curvature with an obstacle. A game theoretical approach.* To appear in Comm. Anal. Geom.