1. Prove that the duration of a coupon bearing bond can not be larger than its time to maturity.

2. Prove, using Itô formula, that the stochastic process

\[ r(t) = \frac{\theta}{\alpha} + e^{-\alpha t} \left[ r_0 - \frac{\theta}{\alpha} \right] + \sigma e^{-\alpha t} \int_0^t e^{\alpha s} dW(s) \]

satisfies Vasicek diffusion equation

\[ dr(t) = (\theta - \alpha r(t)) dt + \sigma dW(t). \]

Based on the first formula, prove that \( r(t) \) is a gaussian random variable, with

\[ \mathbb{E} r(t) = \frac{\theta}{\alpha} + e^{-\alpha t}, \]

\[ \text{var} r(t) = \frac{\sigma^2}{2\alpha} \left[ 1 - e^{-2\alpha t} \right]. \]

Remember that, for a deterministic function \( f(s) \)

\[ \text{var} \int_0^t f(s) dW(s) = \int_0^t f(s)^2 ds. \]