1. (a) Compute the risk-neutral density $q$ for the HSI, if today is June 8, for June 29. Use call option prices as in Lectures 18-19.

(b) Compute the price of a digital european put struck at the money for the same dates. (An European put digital option struck at $K$ pays 1 if $S(T) \leq K$ and 0 if $S(T) > K$.)

2. Prove that for a random variable $T$ with exponential distribution

$$P(T \geq t + h \mid T > t) = P(T > h).$$

Remember that the conditional probability is defined as

$$P(B \mid A) = \frac{P(B \cap A)}{P(A)}.$$

3. Prove that

$$\mathbb{E}_Q \exp \left[ \sigma W(t) \right] = \exp \left[ (\sigma^2/2)t \right].$$

4. Prove that if $Y_1 \sim \mathcal{N}(\nu, \delta^2)$ then

$$\mathbb{E}_Q e^{Y_1} = e^{\nu+\delta^2/2}.$$

5. We want to check by simulation that for a Poisson process with parameter $\alpha$, we have

$$P(N(T) = k) = e^{-\alpha T} \left( \frac{\alpha T}{k!} \right)^k.$$

In order to do this, we choose $\alpha = 1/2$ and $T = 2$. Simulate the jumps of a Poisson process on an interval $[0,2]$ one hundred times. Register the frequencies of the number of jumps obtained in a table, for $k = 0, \ldots, 6$. Compare this register with the theoretical probabilites.