Robust dynamical properties and geometric structures in dimension 3

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Motivation

Theorem (Mañe-Bonatti-Diaz-Pujals-Ures)

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Similar conclusions for stably ergodic diffeomorphisms (Arbieto-Matheus/ Bochi-Fayad-Pujals).

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What about the non-conservative case?

Existence of physical or maximal measures: Sinai-Ruelle-Bowen / Pesin-Sinai/ Alves-Bonatti-Viana/ Burns-Dolgopyat-Pesin/ Vasquez/ Viana-Yang/ Hertz-Hertz-Tahzibi-Ures/ Buzzi-Fisher-Vasquez-Sambarino/ Ures...

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Characterizations of Robust transitivity: ????? Very little is known.

Underlying idea behind this talk

Topological classification can help to understand dynamical consequences.

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Geometric structures give hope that topological restrictions can be found.

Anosov Case:

Theorem (Newhouse-Franks-Manning 70-74)

If $f : N \to N$ is an Anosov diffeomorphism of a infranilmanifold N, then f is topologically conjugate to its linearization (which is also Anosov). Moreover, if $f : M \to M$ and $\dim E^u = 1$ then $M = \mathbb{T}^d$ (and thus conjugate to its linearization which is Anosov).

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So, in dimension ≥ 4 the question seems very hard.

Dimension 2:

Theorem (Pujals-Sambarino)

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In the global dominated splitting case we obtain:

Theorem (Gourmelon-P-Sambarino)

$f : \mathbb{T}^2 \to \mathbb{T}^2$ admitting a global dominated splitting $T\mathbb{T}^2 = E \oplus F$. Then, either f is isotopic to Anosov and f is essentially a DA diffeomorphism or f is essentially Morse-Smale.

We have examples of all cases, and we have studied integrability properties of the bundles to the detail.

Partial hyperbolicity:

From now on, $dim(M) = 3$.

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$f : M \to M$ is *partially hyperbolic* (PH) if $TM = E^{cs} \oplus E^u$ where E^u uniformly expanding and domination between bundles.

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Ergodicity problem: Hertz-Hertz-Ures / Hammerlindl-Ures.

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We say that f is *dynamically coherent* if:

- For f PH, if E^{cs} is integrable to an f-invariant foliation.
- For f SPH, if $E^s \oplus E^c$ and $E^c \oplus E^u$ are integrable to f-invariant foliations ($\Rightarrow E^c$ is also integrable to an f-invariant foliation).

Integrability problem:

In $\mathbb{T}^3$ in the isotopy class of Anosov we contribute to this question:

Theorem (P.)

If $f : \mathbb{T}^3 \to \mathbb{T}^3$ isotopic to Anosov is PH and admits a foliation $\mathcal{F}$ transverse to E^u then f is dynamically coherent.

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- It is an open and closed property.
- It holds for every SPH (Burago-Ivanov).

Classification Problem:

Definition

We say that two dynamically coherent SPH diffeos $f, g : M \to M$ are *leaf conjugate* if there exists $h : M \to M$ homeomorphism such that:

$$h(\mathcal{F}^c_f(f(x))) = \mathcal{F}^c_g(g \circ h(x))$$

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We need *models* to be leaf conjugate to.

Models of SPH diffeos

- Linear Anosov diffeos in $\mathbb{T}^3$ (3 different eigenvalues).
- Skew products over Anosov in $\mathbb{T}^2$ (the manifold is $\mathbb{T}^3$ or non-toral nilmanifold).
- Time one map of Anosov flow ($M =$???).

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Conjecture (Pujals (BW))

If $f : M \to M$ is transitive SPH then (modulo finite lifts and iterate) it is leaf conjugate to one of the above examples.

Transitivity is necessary due to recent example of Hertz-Hertz-Ures. (They conjecture the above changing transitivity for non-existence of *Anosov tori*).

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Conjecture (Hertz-Hertz-Ures)

If $f : M \to M$ has no periodic two-torus tangent to $E^s \oplus E^c$ or $E^c \oplus E^u$ then f is dynamically coherent.

Classification result:

Theorem (Hammerlindl-P)

Let $f : M \to M$ be a SPH without periodic two-torus tangent to $E^s \oplus E^c$ or $E^c \oplus E^u$ and assume that $\pi_1(M)$ is almost solvable. Then, (modulo finite lifts and iterates) it is leaf conjugate to one of the following:

- *A linear Anosov on $\mathbb{T}^3$.*
- *A skew product in $\mathbb{T}^3$ or Nil.*
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The hypothesis on the existence of the periodic torus is to rule out the non-dynamically coherent examples of Hertz-Hertz-Ures which can be done in $\mathbb{T}^3$ (but not in the isotopy class of Anosov) and in solvmanifolds.

Idea of the proof in the Solvmanifold case

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(4) A suitable lift $\tilde{f}$ fixes many leaves of $\tilde{\mathcal{F}}^{cu}_{bran}$ and $\tilde{\mathcal{F}}^{cs}_{bran}$.

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The difficulty is that if it is not coherent, then points can "stop" and possibly both foliations are close to the same.

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(8) There are no periodic points in the universal cover (based on idea of Bonatti-Wilkinson).

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(12) The exponential growth of volume appears in the flow direction, so this allows to perform classical growth arguments to reach a contradiction.

Other manifolds/ Dynamical consequences

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What else???? We need examples.....

Thanks!

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