Dynamics in the study of discrete subgroups of Lie groups

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Context

G a Lie group (can think of $G = \mathrm{GL}(d, \mathbb{R})$ or subgroup).

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If one thinks of Γ as abstract group, the inclusion $\rho : \Gamma \to G$ is a (faithful) *representation*.
Determined by matrices $\rho(a_i) \in G$ (which satisfy the product relations of Γ).

A *deformation* $\rho_t : \Gamma \to G$ is such that $t \mapsto \rho_t(a_i)$ is a continuous path for all $a_i \in S$.

Goals

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- **Rigidity:** Understand which deformations are *genuine* deformations.

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Examples: Stability

- Schottky groups (representations of the free group in k generators),
- Faithful, discrete and cocompact representations (Weil/ Ehresmann-Thurston...)
- Mostow rigidity, Margulis super-rigidity,
- Convex cocompact subgroups of rank 1 Lie groups (includes some Kleinian groups),
- Fuchsian and Quasi-Fuchsian representations (of $\pi_1(\Sigma_g) \to \mathrm{PSL}(2, \mathbb{K}), \ \mathbb{K} = \mathbb{R}, \mathbb{C}$),
- Benoist representations, divisible convexes
- Hitchin representations (of $\pi_1(\Sigma_g) \to \mathrm{PSL}(d, \mathbb{R})$)
- Maximal representations (of $\pi_1(\Sigma_g) \to \mathrm{Sp}(2n, \mathbb{R})$)
- etc.....

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- etc.....

Typically, some kind of *transversality*....

Stability: Link with dynamics

We say $\rho : \Gamma \to G$ is *p-dominated* if $\exists C, \lambda > 0$:

$$\frac{\sigma_p(\rho(\gamma))}{\sigma_{p+1}(\rho(\gamma))} \geq Ce^{\lambda|\gamma|} \qquad \forall \gamma \in \Gamma.$$

$\sigma_i(A)$ denotes the i-th *singular* value of the matrix A if $G = \mathrm{GL}(d, \mathbb{R})$.
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It allows to define *geodesics* in Γ: a geodesic is $\{\gamma_n\}$ such that $|\gamma_m^{-1}\gamma_n| = |n - m|$.

$$\gamma_n = a_{i_1}^{\pm 1} \dots a_{i_n}^{\pm 1} \quad , \quad \gamma_0 = \mathrm{id}$$

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$$\frac{\sigma_p(\rho(\gamma))}{\sigma_{p+1}(\rho(\gamma))} \geq C e^{\lambda|\gamma|} \qquad \forall \gamma \in \Gamma.$$

All geodesics in Γ through id are encoded[2] as sequences of elements of $S \cup S^{-1}$:

$$\Lambda = \{\{a_{i_n}^{\pm 1}\}_{n\in\mathbb{Z}} \ : \ \gamma_n = a_{i_n}^{\pm 1} \dots a_{i_0}^{\pm 1} \text{ geodesic } \} \subset \{a_1^{\pm 1}, \dots, a_n^{\pm 1}\}^{\mathbb{Z}}$$

is a shift invariant closed subset. $T : \Lambda \to \Lambda$.

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is a shift invariant closed subset. $T : \Lambda \to \Lambda$.
One defines a linear cocycle: $A : \Lambda \to G$ as:

$$A(\{x_i\}_{i\in\mathbb{Z}}) = \rho(x_0).$$

[2]When $n < 0$ one needs to adjust the definition of γ_n...

Simple example: Free group in two generators

$\mathbb{F}_2$ free group in two generators u and v.
$G = SL(2, \mathbb{R})$. (Then, $\sigma_1(A) = \sigma_2(A)^{-1} = \|A\|$.)

A sequence $\{x_n\}_{n\in\mathbb{Z}} \in \{u, v, u^{-1}, v^{-1}\}^{\mathbb{Z}}$ belongs to Λ if and only if $x_j \neq x_{j+1}^{-1}$ for all $j \in \mathbb{Z}$.

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If one associates $U = \rho(u) \in G$ and $V = \rho(v) \in G$ then one has that:

$$A(\{x_n\}_n) = U, V, U^{-1} \text{ or } V^{-1} \text{ according to } x_0 = u, v, u^{-1} \text{ or } v^{-1}$$

Domination means that the logarithm of the norm of the product of U, V, U^{-1} and V^{-1} is comparable to the size of the product.

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This is due to a result by Bochi-Gourmelon. Dominated splitting of index p means: for all $\underline{x} \in \Lambda$ there exists a splitting $\mathbb{R}^d = E(\underline{x}) \oplus F(\underline{x})$ with $\dim F(\underline{x}) = p$ such that:

$$A(\underline{x})F(\underline{x}) = F(T(\underline{x})) \qquad \text{and} \qquad A(\underline{x})E(\underline{x}) = E(T(\underline{x}))$$

and there are constants $C, \lambda > 0$ such that for all $v_E \in E(\underline{x}) \setminus \{0\}$ and $v_F \in F(\underline{x}) \setminus \{0\}$ one has:

$$\frac{\|A(T^{n-1}(\underline{x})) \cdots A(\underline{x})v_E\|}{\|v_E\|} \leq Ce^{-\lambda n} \frac{\|A(T^{n-1}(\underline{x})) \cdots A(\underline{x})v_F\|}{\|v_F\|}$$

(Dominated splittings go back at least to Mañé and are still fundamental in the study of differentiable dynamics and linear cocycles.)

Stability: Link with dynamics-Results

Using the cocycle A and properties of dominated splittings we (with J. Bochi and A. Sambarino) give a different proof of the following:

Theorem (Kapovich-Leeb-Porti)

If Γ admits a p-dominated representation then Γ is word hyperbolic.

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Theorem (Kapovich-Leeb-Porti)

If Γ admits a p-dominated representation then Γ is word hyperbolic.

(Cannon) For word hyperbolic groups, (T, Λ) is a *sofic shift* (for simplicity think of shift of finite type).
Based on a criteria of Avila-Bochi-Yoccoz and Bochi-Gourmelon and translating the dominating property into a "dominated splitting" of A we get:

Theorem (joint with J. Bochi and A. Sambarino)

The representation is p-dominated if and only if A admits a familly of invariant multicones only depending on the first coordinate.

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Related results by Kapovich-Leeb-Porti and Guéritaud-Guichard-Kassel-Wienhard (based on notion of *Anosov representations* by Labourie). As a consequence: p-dominated representations are open and the embedding is (stably) *quasi-isometric*.

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Question

If ρ is a stably faithful and discrete representation of a word-hyperbolic group Γ, is it p-dominated for some p?

Partial results by Sullivan, Goldman, Avila-Bochi-Yoccoz, etc.... Related with *stability conjecture* in differentiable dynamics. The question makes sense changing faithful and discrete for quasi-isometric if one assumes $\mathrm{rank}(G) \geq 2$.

Examples: Rigidity

Let $\Gamma_g = \pi_1(\Sigma_g)$ with $g \geq 2$. ($\Gamma_g = \langle a_1, b_1, \dots, a_g, b_g \mid \prod_i [a_i, b_i] = \mathrm{id} \rangle$)

Theorem (Bowen)

Let $\rho : \Gamma_g \to \mathrm{PSL}(2, \mathbb{C})$ quasi-isometric representation[a]. Then $\dim_H(\rho(\partial_\infty \Gamma)) \geq 1$ and equality holds if and only if $\rho(\partial_\infty \Gamma)$ is a round circle (i.e. ρ is Fuchsian).

[a] Called *quasi-Fuchsian*.

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Can be rephrased in terms of *critical exponent*:

$$h_\rho = \lim_T \frac{1}{T} \log \sharp\{\gamma \in \Gamma \ : \ \log \|\rho(\gamma)\| \leq T\}$$

Sullivan showed $h_\rho = \dim_H(\rho(\partial_\infty \Gamma))$ (using Patterson-Sullivan measures).

Examples: Rigidity

Idea of the proof:

- Construct a hyperbolic dynamical system ϕ where $\dim_H(\rho(\partial_\infty \Gamma)) = h_{top}(\phi)$.
- Use thermodynamic formalism to obtain rigid inequalities

One has $h_\rho = \dim_H(\rho(\partial_\infty \Gamma)) = h_{top}(\phi)$.

Examples: Rigidity

Other examples:

- In rank one: Besson-Courtois-Gallot, Bourdon.....
- Higher rank: Bishop-Steger, Burger, Crampon......

I will present a rigidity result obtained joint with A. Sambarino for *Hitchin representations*.

Hitchin representations

Consider a Fuchsian representation ρ_F of $\Gamma = \pi_1(\Sigma)$ into $\mathrm{PSL}(2,\mathbb{R})$. Let $\imath : \mathrm{PSL}(2,\mathbb{R}) \to \mathrm{PSL}(d,\mathbb{R})$ an *irreducible* representation.[3]

The connected component of $\rho_0 = \imath \circ \rho_F$ is called the *Hitchin component* denoted by $\mathrm{Hit}(\Gamma, d)$. Representations fixing a copy of $\mathrm{PSL}(2,\mathbb{R})$ as ρ_0 are in the *Fuchsian locus* of the component.

[3] i.e. with no invariant subspaces; it is unique up to conjugacy.

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(Hitchin, Labourie, Fock-Goncharov, Guichard-Wienhard, etc...) Teichmuller theory in higher rank (*Higher Teichmuller Theory*):

- Every $\rho \in \mathrm{Hit}(\Gamma, d)$ is faithful and discrete.
- If $\rho \in \mathrm{Hit}(\Gamma, d)$ then $\rho(\gamma)$ is *loxodromic* (all eigenvalues are diferent) for every $\gamma \in \Gamma$.

Also, there are several geometric interpretations of these representations for the surfaces.

[3] i.e. with no invariant subspaces; it is unique up to conjugacy.

Critical exponent

As before, we define, for $\rho \in \mathrm{Hit}(\Gamma, d)$, its *critical exponent*:

$$h_\rho = \limsup_T \frac{1}{T} \log \sharp\{\gamma \in \Gamma \; : \; \log \|\rho(\gamma)\| \leq T\}$$

We let $\alpha = h_{\rho_F}$ where ρ_F is Fuchsian. Notice that for a Fuchsian representation this always takes the same value (Gauss-Bonnet).

A rigidity result

Theorem (joint with A. Sambarino)

One has that $h_\rho \leq \alpha$ and $h_\rho = \alpha$ if and only if ρ is in the Fuchsian locus.

Some ideas of the proof: Counting eigenvalues in different directions

Let $\mathcal{L}_\rho$, the *limit cone*. The smallest closed cone in $\mathfrak{a}^+ = \{(a_1, \ldots, a_d) \ : \ a_1 + \ldots + a_d = 0 \ , \ a_1 \geq a_2 \geq \ldots \geq a_d\}$ containing the eigenvalues $\lambda(\rho(\gamma))$ of matrices $\rho(\gamma)$ with $\gamma \in \Gamma$. (Benoist, Quint, Sambarino....).

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If $\varphi \in \mathfrak{a}^*$ is positive in $\mathcal{L}_\rho$ one can define its *entropy*

$$h_\rho^\varphi = \limsup_T \frac{1}{T} \log \sharp\{[\gamma] \in [\Gamma] \ : \ \log \varphi(\lambda(\rho(\gamma))) \leq T\}$$

Sambarino showed that $h_\rho^\varphi \in (0, +\infty)$ and clearly $h_\rho^{t\varphi} = \frac{1}{t} h_\rho^\varphi$. One defines $\mathcal{D}_\rho = \{\varphi \ : \ h_\rho^\varphi \leq 1\} \subset \mathfrak{a}^*$. It is related with the entropy of a certain reparametrization of the geodesic flow (which is Anosov) and some results of Ledrappier.

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One defines $\mathcal{D}_\rho = \{\varphi \ : \ h_\rho^\varphi \leq 1\} \subset \mathfrak{a}^*$.
Using deep properties of the thermodynamic formalism one deduces that $\partial\mathcal{D}_\rho$ is a convex analytic submanifold of codimension 1 in $\mathfrak{a}^*$ (Sambarino).

Some ideas of the proof: Blocking entropy gives rigidity

Fact: (Quint-Sambarino)[4] $h_\rho = d_{\mathfrak{a}^*}(0, \partial\mathcal{D}_\rho)$.

[4] Here $d_{\mathfrak{a}^*}$ is a distance in $\mathfrak{a}^*$ which is independent of ρ. It can be made explicit but we will ignore this.

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Fact: (Quint-Sambarino)[4] $h_\rho = d_{\mathfrak{a}^*}(0, \partial\mathcal{D}_\rho)$.

If we have information on h_ρ^φ for some φ we have information on h_ρ.

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Fact: (Quint-Sambarino)[4] $h_\rho = d_{\mathfrak{a}^*}(0, \partial \mathcal{D}_\rho)$.

If we have information on h_ρ^φ for some φ we have information on h_ρ.

Moreover, if we *block* some $\varphi_i \in \mathfrak{a}^*$ which we know will *always* belong to $\mathcal{D}_\rho$, then we will know that $h_\rho = d(0, \partial \mathcal{D}_\rho)$ is bounded by the distance to the affine hyperspace generated by the φ_i.

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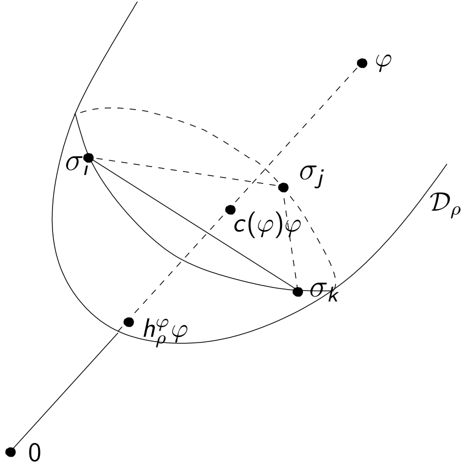

Figure: The functionals σ_i have entropy one for all ρ.

Fact: (Quint-Sambarino)[5] $h_\rho = d_{\mathfrak{a}^*}(0, \partial\mathcal{D}_\rho)$.

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If the distance is attained then $\partial\mathcal{D}_\rho$ coincides with the affine hyperspace (analyticity) and we get rigidity. (Benoist: if there are algebraic relations between the eigenvalues, then the Zariski closure is *smaller*.)

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Remark: Bridgeman-Canary-Labourie-Sambarino showed that the functions $\rho \mapsto h_\rho^\varphi$ are *analytic*. Then it is enough to block them in a neighborhood of the Fuchsian locus.

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Some ideas of the proof: SRB measures

Theorem (joint with A. Sambarino)

The entropy of $\eta_i(a_1, \ldots, a_d) = a_i - a_{i-1}$ *is equal to* 1 *in a neighborhood of the Fuchsian locus.*

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Very rough idea: Construct a 3-dimensional $C^{1+\alpha}$-Anosov flow (orbit equivalent to the geodesic flow in the surface) whose unstable jacobian measures exactly η_i in periodic orbits (which correspond to conjugacy classes of elements in Γ). A Theorem by Sinai-Ruelle-Bowen implies that the reparametrization by η_i has entropy equal to 1. Which provides the result.

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Remark: The $C^{1+\alpha}$ hypothesis is crucial and the technical core of the proof is to establish it. The key tool to construct the flow are the limit maps of the representation which correspond essentially to the bundles in the dominated splitting.

Some questions

- Are there other critical points of the critical exponent? Local maxima different from Fuchsian locus?
- Why in rank 1 the critical exponent has a minimum while in higher rank it is a maxima?
- Is the regularity of the Anosov flow we construct related to some kind of *normal hyperbolicity*?

Thanks!

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