IMPA - Int'l Congress in dynamical Systems | Geometry of the strong foliations in partially hyperbolic attractors. | 28 Nov. 2013

$f: M \to M$ C^1-diffeomorphism. $\Lambda \subseteq M$ compact f-invariant set

Λ is partially hyperbolic: $T_\Lambda M = E^s \oplus E^c \oplus E^u$

Recall: $\exists\, W^s$ & W^u stable & unstable laminations

We shall study Λ such that is saturated by W^u.

[Examples: → Λ is an attractor (not. nec. transitive): $\Lambda = \bigcap_{n \geq 0} f^n(\bar{U})$ where $f(\bar{U}) \subseteq U$.

→ Λ is a quasi-attractor: Λ is a (at most countable) intersection of attractors which is chain-recurrent.

→ Λ is a Haussdorf limit of attractors/quasi-attractors]

Goal:. Study the geometry of minimal W^u-saturated sets.

↳ minimal for: → Compact f-invariant
→ W^u-saturated

. Finiteness?

Finiteness of attractors, particularly far from tangencies is one of the main themes in Palis' Conjectures.

[Conjecture (Tameness Conjecture - Bonatti) A C^1-generic diffeomorphism far from homoclinic tangencies has finitely chain-recurrence classes.]

Pujals - Sambarino - Wen - Crovisier - D & J. Yang $\Big\}$ ⇒ Partial hyperbolicity with one-dim central bundles is abundant far from tangencies.

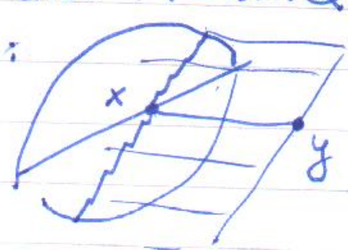
Thm (CPS) f is C^1-generic far from homoclinic tangencies $\Rightarrow$ finitely many sinks/sources (No-Newhouse Phenomena)

↳ Bonatti-Diaz ($\dim M \geq 3$)

Thm (BGLY) f is C^1-generic far from homoclinic tangencies $\Rightarrow$ quasi-attractors are isolated from each other.

In principle they may accumulate: That is why we study Hausdorff limits of quasi-attractors.

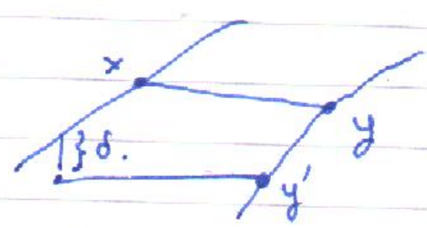
Idea: If $\dim E^c = 1$ and $\Lambda = \lim Q_n$ has points in the following configuration:



$\Rightarrow$ the Q_n intersect each other $\Rightarrow$ Contradiction.

Thm (joint with S. Crovisier & M. Sambarino) $\exists\, \mathcal{U} \subseteq \mathrm{Diff}^1(M)$ C^1-open and dense such that if $f \in \mathcal{U}$ and $\Lambda \subseteq M$ is a W^u-saturated p.h. set $\Rightarrow$ $\exists\, \delta > 0$ and $L > 0$ verifying: If $x, y \in W^s(x) \cap \Lambda$ and $d_s(x,y) \in [\|Df\|^{-1} L, L]$ then, $\exists\, y' \in W^u_L(y)$ such that
$$d(W^s_{2L}(y'), W^u_{2L}(x)) > \delta.$$

Uniform non joint integrability.



Main Difficulty (vs DW): Control the continuation of Λ.

Related: Abdenur-Crovisier: C^1-Generic transitive globally p.h. diffeos with $\dim E^c = 1$ are Robustly transitive.

Some Consequences of our Result:

Consequence 1: $\exists\, \mathcal{U} \subseteq \mathrm{Diff}^1(M)$ open & dense such that if Λ is a $\mathcal{W}^u$-saturated p.h. set of $f \in \mathcal{U}$ with $\dim E^c = 1 \Rightarrow \Lambda$ contains finitely many $\mathcal{W}^u$-saturated sets.

Corollary: $\exists\, \mathcal{U} \subseteq \mathrm{Diff}^1(M^3) \setminus \overline{\mathrm{Tang}}$ open & dense such that if $f \in \mathcal{U}$ $\Rightarrow f$ has finitely many quasi-attractors.

SOME IDEAS OF THE PROOFS:

I) Assuming the Theorem:

Idea: Consider Λ_n $\mathcal{W}^u$-saturated disjoint minimal sets (or Q.A.) and $\Lambda = \lim_H \Lambda_n$ which is also $\mathcal{W}^u$-saturated.

Possibilities: $\rightarrow \nexists\, x, y \in \Lambda$ in the same strong-stable
Then, BC + PS $\Rightarrow \Lambda$ is hyperbolic $\Rightarrow$ isolated ✓.

$\rightarrow \exists\, y \in W^s(x) \cap \Lambda$, two possibilities:

$\rightarrow$ Transverse $\Rightarrow$ finiteness.
$\rightarrow$ non-transverse: More carefull analysis is needed.

KEY IDEA: ~~look~~ Apply the thm for the Λ_n's: Either one has points in the same stable ($\Rightarrow$ one constructs open sets of large diameter and "good" properties) or Λ_n's are hyperbolic (& codimension one in the center $\Rightarrow$ large in "one side"...).

II) PERTURBATION RESULT: How to control the continuation?

Reason for C^1-perturbations: → The first one already appears in DW, one wants that the perturbation is in a small region disjoint from many iterates: One moves W^u without changing W^s too much.

→ One must control the continuation: One must make perturbations in plenty of places so that one "captures" every point in Λ.

<u>Idea</u>: 1) Cover the manifold by sections of W^u which are sufficiently wandering (disjoint from a lot of iterates)

This allows to change one bundle without changing the other.

2) At a small scale, the E^u-bundle (before perturbation) is almost linear one uses this to guarantee that "after" perturbation, the bundles are almost "parallel" at that scale in the E^s-direction.

3) One divides the place into regions, some will be perturbed, some not, and this gives the breakeage of joint integrability.