Geometric Flows and Relativity

March 18, 19, 20 - Punta del Este, Uruguay

Talks

José Espinar

(Universidad de Granada, Spain)

Title: An overdetermined eigenvalue problem and the Critical Catenoid conjecture

Abstract: We consider the eigenvalue problem $\Delta^2\xi + 2\xi = 0$ in Ω and $\xi = 0$ along $\partial\Omega$, being Ω the complement of a disjoint and finite union of smooth and bounded simply connected regions in the two-sphere $\mathbb{S}^2$. Imposing that $|\nabla\xi|$ is locally constant along $\partial\Omega$ and that ξ has infinitely many maximum points, we are able to classify positive solutions as the rotationally symmetric ones. As a consequence, we obtain a characterization of the critical catenoid as the only embedded free boundary minimal annulus in the unit ball whose support function has infinitely many critical points.

Zhongshan An

(University of Michigan, USA)

Title: Einstein metrics on manifolds with boundary

Abstract: We will talk about existence of Einstein metrics on manifolds with boundary, while prescribing the conformal class of the induced metric and mean curvature of the boundary. In dimension 3, this becomes the existence of conformal embeddings of surfaces into constant sectional curvature space forms, with prescribed mean curvature. We will discuss the local existence results near umbilic round spheres and non-umbilic star-shaped surfaces. Furthermore, for higher dimensions, we will introduce a non-degenerate boundary condition which will guarantee the local existence.

Francisco Vanderson Moreira De Lima

(Universidade Federal do Rio Grande do Sul, Brasil)

Title: Eigenvalue problems and free-boundary minimal surfaces in spherical caps.

Abstract: In this talk, I will present a family of functionals on the space of Riemannian metrics of a compact surface with boundary, defined via eigenvalues of a Steklov-type problem, whose maximizing metrics are induced by free boundary minimal immersions in some geodesic ball of a round sphere. Also, I will discuss the maximizer in the case of a disk and present a characterization of a certain class of annuli. This is joint work with Ana Menezes (Princeton University).

Marcos Petrúcio Cavalcante
(Universidade Federal de Alagoas)

Title: New solutions to the Serrin problem on cohomogeneity one manifolds.

Abstract: The Serrin problem is an overdetermined elliptic problem that shares many analogies with constant mean curvature surfaces. In this talk, we will revisit some known results related to the existence and classification of solutions to the Serrin problem on certain Riemannian manifolds. Motivated by these works, we will use bifurcation theory to prove the existence of non-trivial domains that admit solutions to the Serrin problem on cohomogeneity one Riemannian manifolds. This is joint work with Renato Bettiol (CUNY).

Sebastien Alvarez
(Universidad de la República, Uruguay)

Title: Asymptotic counting of surface subgroups and foliated Plateau problems.

Abstract: We study the dynamical properties of the space of k-surfaces, that is, suitably complete immersed surfaces of constant extrinsic curvature in negatively curved 3-manifolds,. This study was initiated in the late 90s by Labourie, who presented these objects as a higher-dimensional analogue of the geodesic flow. In this talk, following the recent work of Calegari–Marques–Neves, we study the asymptotic counting of surface subgroups in terms of areas of k-surfaces. We determine a lower bound, and we prove rigidity when this bound is achieved. This is joint work with Ben Lowe (University of Chicago) and Graham Smith (PUC Rio de Janeiro).

Marcus Khuri
(State University of New York at Stony Brook)

Title: Black Holes of Lens Space Topology

Abstract: We present the first examples of formally asymptotically flat black hole solutions with horizons of general lens space topology L(p,q). These 5-dimensional static/stationary spacetimes are regular on and outside the event horizon for any choice of relatively prime integers $1 \leq q < p$, in particular conical singularities are absent.They are supported by Kaluza-Klein matter fields arising from higher dimensional vacuum solutions through reduction on tori. The technique is sufficiently robust that it leads to the explicit construction of regular solutions, in any dimension, realising the full range of possible topologies for the horizon as well as the domain of outer communication, that are allowable with multi-axisymmetry. Lastly, as a by product, we obtain new examples of regular gravitational instantons in higher dimensions.

Brett Kotschwar
(Arizona State University)

Title: Ricci flows which terminate in cones.

Abstract: Asymptotically conical shrinking Ricci solitons give rise to solutions to the Ricci flow which flow smoothly into the end of a cone at the singular time. We will show that this phenomenon is peculiar to shrinking solitons: any complete solution to the Ricci flow on $M \times [-T, 0)$ which has quadratic curvature decay on some end of M and converges locally smoothly to the end of a cone on that neighborhood as $t \nearrow 0$ must be globally a gradient shrinking soliton at times $t < 0$.

Pedro Gaspar
(Pontificia Universidad Católica de Chile)

Title: A Morse-theoretic glance at phase transitions approximations of mean curvature flows

Abstract: The Allen–Cahn equation is a semilinear parabolic partial differential equation that models phase-separation phenomena and which provides a regularization for the mean curvature flow of hypersurfaces. In this talk, we employ Morse-theoretical considerations to construct eternal solutions of the Allen–Cahn equation that connect unstable stationary solutions in compact manifolds. We describe the space of such solutions in a round 3-sphere under a low-energy assumption, and indicate how these solutions can be used to produce geometrically interesting eternal MCFs. This is joint work with Jingwen Chen (University of Pennsylvania).

Julian Scheuer
(Goethe Universität Frankfurt)

Title: Foliations of null hypersurfaces by surfaces of constant curvature near MOTS

Abstract: Recently, Henri Roesch and myself have studied a mean curvature flow in null hypersurfaces to prove the existence of MOTS under some relatively weak assumptions on the null hypersurface. In a continuation of this approach, together with Wilhelm Klingenberg and Ben Lambert we prove the existence of foliations of null hypersurfaces near MOTS by certain constant curvature surfaces. In this talk we have a look at a modified null mean curvature flow to construct such foliations.

Lucas Ambrozio
(IMPA)

Title: Why are Zoll metrics interesting?

Abstract: In the beginning of the last century, Otto Zoll discovered smooth, rotationally symmetric spheres in the Euclidean space, not the round ones, which have the remarkable property that all of their non-trivial geodesics are periodic, simple and have the same length. Since then, a challenge has been to classify compact Riemannian manifolds whose geodesic flow has this property. Besides the obvious geometric interest of such curious metrics, it is interesting to observe that Zoll metrics admit geometric-variational characterisations. This lead to the curious possibility of investigating "Zoll phenomena" in other variational theories. This talk will be about this circle of ideas, and will be based on recent joint work with some of our collaborators (F. Marques, A. Neves, R. Montezuma and R. Santos).

Leandro Del Pezzo
(Universidad de la República, Uruguay)

Title: Fractional convexity

Abstract: In this talk, we introduce a notion of fractional convexity that extends naturally the usual notion of convexity in the Euclidean space to a fractional setting. With this notion of fractional convexity, we study the fractional convex envelope inside a domain of an exterior datum (the largest possible fractional convex function inside the domain that is below the datum outside) and show that the fractional convex envelope is characterized as a viscosity solution to a non-local equation that is given by the infimum among all possible directions of the 1-dimensional fractional Laplacian. In addition, we show that the fractional convex envelope converges as $s \uparrow 1$ to the classical convex envelope. The results of the talk have been obtained in collaboration with B. Barrios (ULL), A. Quaas (USM) and J. Rossi (UBA).