

# THE RICCI FLOW — A MINICOURSE

SUMMER SCHOOL ON GEOMETRIC FLOWS AND RELATIVITY, UNIVERSIDAD  
DE LA REPÚBLICA, MONTEVIDEO

MAT LANGFORD

## COURSE OUTLINE

The following list gives a rough idea of the topics we hope to cover (to varying degrees of depth).

- Symmetry properties and special solutions. Short time existence. The Uhlenbeck trick. Evolution equations. The (tensor) maximum principle.
- Shi’s (Bernstein) estimates. Long time existence. Blow-ups and compactness.
- Ricci flow on surfaces.
- Three-manifolds with positive Ricci curvature. Manifolds with positive curvature operator. The 1/4-pinched differentiable sphere theorem. Pinched manifolds are compact.
- Curvature pinches towards positive, the differential Harnack inequality, no local collapsing.
- Survey of attempts at the classification of ancient solutions.

## PREREQUISITES

Background in the foundations of Riemannian geometry is expected. Some background in the foundations of elliptic and parabolic partial differential equations is desirable.

## REFERENCE MATERIAL

There are now very many good books and lecture notes on Ricci flow. For an introduction, we recommend, in particular, [\[1, 2, 3, 4\]](#)

## REFERENCES

- [1] Ben Andrews and Christopher Hopper. *The Ricci flow in Riemannian geometry*, volume 2011 of *Lecture Notes in Mathematics*. Springer, Heidelberg, 2011. A complete proof of the differentiable 1/4-pinching sphere theorem.
- [2] Bennett Chow, Sun-Chin Chu, David Glickenstein, Christine Guenther, James Isenberg, Tom Ivey, Dan Knopf, Peng Lu, Feng Luo, and Lei Ni. *The Ricci flow: techniques and applications. Part I*, volume 135 of *Mathematical Surveys and Monographs*. American Mathematical Society, Providence, RI, 2007. Geometric aspects.
- [3] Bennett Chow and Dan Knopf. *The Ricci flow: an introduction*, volume 110 of *Mathematical Surveys and Monographs*. American Mathematical Society, Providence, RI, 2004.
- [4] Bennett Chow, Peng Lu, and Lei Ni. *Hamilton’s Ricci flow*, volume 77 of *Graduate Studies in Mathematics*. American Mathematical Society, Providence, RI; Science Press Beijing, New York, 2006.