2. Predictability of asset returns*

Deterministic and Random Walk approach

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Question:

Can we predict future values of an asset?

More precisely:

How to model the time evolution of prices of financial instruments, as stocks, indexes, commodities, etc.

We have different approachs:

- Deterministic Approach (YES, we can predict)

- Stochastic Approach:
  - Unpredictable (random walk - martingale, NO, we can not predict)
  - Predictable (time series, YES, we can statistically predict)

2a. Deterministic approach

Consists in the assumption that future prices, future demand, etc. are deterministic, i.e. precisely known in advance. The question is then to make the financial decisions in order to maximize the gain of investors.

For instance, in order to decide whether to invest in a project that costs I_0, one computes the Net Present Value (NPV) of this investment as

$$NPV = \sum_{t=1}^{n} \frac{S_t}{(1+k)^t} - I_0$$

where S_t is the expected income of year t, I_0 the initial investment, k is the discount rate, and n is the duration (in years) of the project.

2b. Stochastic Approach

Initiated by Markowitz (1952) who considered the optimization of investment decisions under uncertainty, the so called mean-variance analysis, with the main consequence of the advantages of diversification in order to reduce the risk, measured here as the variance of the expected return.

A second step was the Capital Asset Pricing Model (CAPM) by Sharpe (1964), constructing an optimal invesmentment policy in a portfolio with several different assets.

A third step was the Arbitrage pricing theory (APT) of Ross (1976), being a generalization of the latter.

In reference to the risks, we distinguish

- Unsystematic risks that can be reduced, for instance, by diversification,

- Systematic risks due to the stochastic nature of the models.

2c. Unpredictability of asset returns:

- **Efficient markets**

- **Random walk hypothesis**

An efficient market has:

- instantaneous corrections of prices withouth posibility of buying low to sell high (i.e. we do not have arbitrage opportunities).

- All dealers interpret price movements in the same way

- The participants are homogeneous and behave rationally

This three postulates are consistent with the Random walk hypothesis, that proposes to model prices of an asset $\{S_n\}$ as

$$S_n = \exp(h_1 + \cdots + h_n), \qquad n \geq 1,$$

where $h_1, h_2, \ldots$ is a sequence of independent random variables, defined in a probability space $(\Omega, \mathcal{F}, \mathbf{P})$.

In other words, the random variables

$$h_1 = \ln \frac{S_1}{S_0}, \ldots, h_n = \ln \frac{S_n}{S_{n-1}}, \ldots$$

are independent.

Several empirical studies confirm this hypothesis in different situations (stock markets, commodity prices) but of course, one should be cautious about the universality of this model.

In continuos time, Samuelson (1965) and Black and Scholes (1973) and Merton (1973) proposed the model

$$S_t = S_0 \exp\Big(\sigma W_t + (\mu - \sigma^2/2)t\Big),$$

for $t \geq 0$, where

- $\{W_t\}$ a Wiener process defined on $(\Omega, \mathcal{F}, \mathbf{P})$,
- σ is the volatility,
- μ is the expected return.