Statistical Methods and Calibration in Finance and Insurance - MA6622

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Course details:

- 25 lectures (50 + 10 min.)

- corresponding assignments

- 14 tutorials (50 + 10 min.)

Course Assessment:

- 40 % course-work: indicated exercises in assignments, in groups (2 or 3 students).

- 60 % examination (July 25 from 18:30 to 21.30)

Introduction*

1. Financial structures and instruments

1a. Key objects and structures

When mathematically modelling financial activities, we consider

- individuals

- corporations

- intermediaries

- financial markets

By individuals we mean agents that behave according to the principle of consumption and investment maximization

By corporations we mean companies or firms that own land, factories and/or machines, and organize business and manufacturing.

By intermediaries we mean intermediate financial structures, like banks, investments companies, pensions funds, insurance companies, etc. and also the stock exchanges, options and futures exchanges.

By financial markets we mean money and Foreign exchange (Forex) markets, and markets of financial instruments, including financial instruments and securities.

We are specially interested in the description of financial markets.

1b. Financial Markets

In the financial market one distinguishes:

(A) underlying or primary instruments, like

- bank accounts

- bonds

- stocks

(B) derivative or secondary instruments, like

- options

- futures, warrants, swaps, etc.

By financial engineering we mean manipulation of derivative securities, in order to increase capital and/or reduce risk.

Financial markets

Main ingredients of financial markets are:

(1) Money, that emerged as a way of exchanging "things one has" by "things one can get". We say local or domestic currency to mean Hong Kong Dollars (HKD).

(2) Foreign currency that is the currency of other nations, as US dollars, Euros, British Pounds, etc.

(3) Precious metals that played and still play an importat role, first when money was convertible into gold, and now as a way to mantain reserves

(4) Bank account is similar to a bond (see (5)), that reduces to the obligation of a bank to pay certain interest on the sum put into one's account. Bank account is usually a "unit of measurement" or "numeraire", for the prices of other securities.

In what concerns the interest rate, we can have

- simple interest rate $r(m)$ payed m times a year or

- continous interest rate $r(\infty)$.

After N years with an initial capital B_0 the account has

$$B_N(m) = B_0\Big(1 + \frac{r(m)}{m}\Big)^{mN}$$

in the first case, and in the second

$$B_N(\infty) = B_0 \exp\Big(r(\infty)N\Big).$$

(5) Bonds are promisory notes issued by a government, bank or corporation, or other financial establishment to raise capital. When issued at time $t = 0$ a bond is characterized by

- the face value $P(T, T)$, i.e. the sum payable to the holder at the

- maturity date T (that can be from 2 to 30 years)

- the interest rate r_c defining the amount payable to its holder by the issuer (the dividends) by the formula $r_c \times P(T,T)$ each year.

- the original price $P(0,T)$ of the bond with maturity T years, issued at time $t = 0$.

- the market value $P(t,T)$ at time $t \in [0,T]$

- the current yield given by

$$r_c(t, T) = \frac{r_c \times P(T, T)}{P(t, T)}, \qquad 0 < t < T.$$

- the yield to maturity or profitability (on a percentage basis), denoted by

$$\rho = \rho(T - t, T)$$

computed as the root of the equation

$$P(t, T) = \sum_{k=1}^{T-t} \frac{r_c P(T, T)}{(1 + \rho)^k} + \frac{P(T, T)}{(1 + \rho)^{T-t}}$$

(6) Stocks are issued by companies to raise funds. They can be ordinary shares (equities), or preference stocks with difference in the riskiness and the dividends policy. Stocks are bought by individuals, but mainly by "institutional investors" as funds, banks, etc, in intermediation institutions as the New York Stock Exchange (NYSE) and the Hong Kong Stock Exchange (HKSE).

In order to see the "global state" of the economy, we have indexes that ponderate the prices of a group of stocks, as the "Standard & Poors 500", NASDAQ, or here the "Hang Seng Index (HSI)", ponderating the evolution of stock prices of the 33 largest companies of the Hong Kong stock market.

1c. Derivatives and financial instruments

- Options that give the right to buy or sell something at a prescribed excercies price K at a fixed excercise time T or during a prescribed interval of time $[0, T]$.

	Fixed time T	Interval $[0, T]$
right to buy	European Call	American Call
right to sell	European Put	American Put

Option's denomination according to buy/sell and excercise time

- **Futures** that is a compromise to buy or sell something at a fixed price, in a prescribed date.

- **Exotic Options** . . .

Main difference between options and futures: if you have an option you can refuse to buy, otherwise, if you signed a futures contract, you **should** buy at the prescribed price.