Statistical Methods and Calibration in Finance and Actuarial Science

MA6622 Assignment of Lectures 18 to 21

1. (a) Compute the risk-neutral density q for the HSI, if today is June 8, for June 29. Use call option prices as in Lectures 18-19.

(b) Compute the price of a digital european put struck at the money for the same dates. (An European put digital option struck at K pays 1 if $S(T) \leq K$ and 0 if $S(T) > K$.)

2. Prove that for a random variable T with exponential distribution

$$\mathbf{P}(T \geq t + h \mid T > t) = \mathbf{P}(T > h).$$

Remember that the conditional probability is defined as

$$\mathbf{P}(B \mid A) = \frac{\mathbf{P}(B \cap A)}{\mathbf{P}(A)}.$$

3. Prove that

$$\mathbf{E_Q} \exp\Big[\sigma W(t)\Big] = \exp\left[(\sigma^2/2)t\right].$$

4. Prove that if $Y_1 \sim \mathcal{N}(\nu, \delta^2)$ then

$$\mathbf{E_Q}\, e^{Y_1} = e^{\nu + \delta^2/2}.$$

5. We want to check by simulation that for a Poisson process with parameter α, we have

$$\mathbf{P}(N(T) = k) = e^{-\alpha T}\left(\frac{\alpha T}{k!}\right)^k.$$

In order to do this, we choose $\alpha = 1/2$ and $T = 2$. Simulate the jumps of a Poisson process on an interval $[0, 2]$ one hundred times. Register the frequencies of the number of jumps obtained in a table, for $k = 0, \ldots, 6$. Compare this register with the theoretical probabilites.