**Statistical Methods and Calibration in Finance and Actuarial Science**

MA6622 Assignment of Lectures 14-15

**1.** Obtain the Black Scholes Formula departing from

$$C(S(0), K, T, r, \sigma) = \mathbf{E}_{\mathbf{Q}}\, e^{-rT}\big(S(T) - K\big)^+$$

**2.** Departing from the quotations of call options written on the the HSI, compute first the risk-free interest rate, departing from the future price with the same expiration, and check that the implied volatiliy coincides with the quoted one.

**3.** Compute the price of an option struck at 15000, but expiring on August 10. Perform the following steps:

Step 1 Compute from the futures price the riskfree interest rates for the end of July and the end of August. Take for computing the price the ponderated mean of this risk free rates, taking into account the number of trading days before and after July 10.

Step 2 Compute from implied volatiliy from the option prices quotations for the July and August Call Option. Take the same ponderation to compute the volatility to be used in Black-Schoes formula.

Step 3 Compute the price of the option with BS formula.

Compute also the ponderated means of the option prices. Are they significatively different? In case they are different, do you have an explanation for this difference?

**4.** Usually prices on HSI options are qouted for strikes every 200 points. Compute the price of an options struck at the mid point of two given strikes, for nearly at the money values, expiring for instance, in August. Use quotations from some current newspaper, and linearly interpolate the volatility. Compare the price obtained with the linear interpolation of prices.

**5.** We say that a random variable $X$ has a Bernoulli distribution with parameter $p$ when it takes the values

$$X = \begin{cases} u - 1 & \text{with probability } p \\ d - 1 & \text{with probability } 1 - p \end{cases}$$

Compute the expectation $\mathbf{E}\,X$ and the variance $\mathbf{var}\,X$ of a Bernoulli r.v.

**6.** We say that a random variable $X$ has a Binomial distribution with parameters $n, p$ when it takes the values

$$S = i \quad \text{with probability } C_i^n p^i (1-p)^{n-i} \text{ where } i = 0, \dots, n.$$

(a) Prove that if $X_1, \dots, X_n$ are independent random variables with common Bernoulli distribution with parameter $p$, then

$$S = X_1 + \cdots + X_n$$

has a Binomial distribution with parameters $n, p$.

(b) Using the previous result, compute compute the expectation $\mathbf{E}\,S$ and the variance $\mathbf{var}\,S$ of a Binomial random variable.

**7.** Compute the price of an American Put Option written on the HSI, taking

$$S(0) = 15248,\ K = 15000,\ T = 32/247,\ r = 0.025,\ \sigma = 0.22.$$

using the method of backwards induction with 32 steps.

**8.** Explain why the early excercise premium of an American Option is always positive.

**9.** Comment the quotation "The higher the volatility of the underlying stock, the more likely it will be for both call and put options to become in-the-money, and consequently their values will be higher." from the FAQ at the Hong Kong Stock Exchange Web Page

http://www.hkex.com.hk/invedu/faq/prod_der_stopt.htm