

## Statistical Methods and Calibration in Finance and Insurance

MA6622

Assignment of Lectures 10-11

For the assessment you can choose: Exercise 3, Exercise 6, or both. (Deadline: Friday 14th July).

1. Assume that you have a register with  $N$  profit and loss  $PL$  records, in order to estimate the historical VaR of a portfolio. Determine the number of values (after ordering them in increasingly) smaller than  $-\text{VaR}$  for confidence levels  $\alpha = 0.95$  and  $\alpha = 0.99$  in the following cases:

- $N = 100, N = 250, N = 360, N = 1000$ .

2. Read the article in Wikipedia about Basel II.

[http://en.wikipedia.org/wiki/Basel\\_II](http://en.wikipedia.org/wiki/Basel_II).

Find more information (if necessary) to answer the questions:

- What is the purpose of Basel II.
- Which are the three pillars of Basel II, and what do they mean.
- Do Basel II force financial institutions (Banks in this case) to use a certain model to compute VaR, or can these institutions choose their model?
- In case financial institutions can choose their models, do you think this is an important fact?

3. Compute the VaR of the HSI for 10 days, computing the 5% quantile of the empirical distribution, using the last year of daily data. Compare with the Normal and Lognormal calculations of Lecture 10. Explain (if found) the differences.

4. Compute the VaR of your portfolio of Exercise 3 (Assignment 4-5) for 10 days with one year of daily data, computing the 5% quantile of the empirical distribution.

5. Compute VaR for a portfolio  $V_\pi$ , for a 1 day period, through simulation. Assume that the logarithmic returns

$$X(t) = \log \left( \frac{S(t)}{S(t-1)} \right),$$

follow an AR(1) model

$$X(t) = \frac{1}{4}X(t-1) + \varepsilon(t),$$

with  $\{\varepsilon(t)\}$  a strict white noise with mixed normal distribution, with density

$$f_\varepsilon(x) = \frac{1}{4}\varphi\left(\frac{x}{\sqrt{2}}\right) + \frac{3}{4}\varphi\left(\frac{x}{\sqrt{2/3}}\right)$$

where

$$\varphi(x) = \frac{1}{\sqrt{2\pi}}e^{-x^2/2}$$

is the density of a standard normal random variable.

Perform 100 time steps in order to reach the stationary distribution for  $X(t)$ ; repeat the simulation experiment 1000 times in order to properly estimate the density of  $V(t) = V(0) \exp[X(t)]$ .

**6.** The objective is to estimate the VaR of a portfolio, with 1 year horizon, that comprises two assets:

- A bond  $A$  with  $\mu_A = 0.08$  expected annual log-return, and  $\sigma_A = 0.10$  standard deviation,
- A stock with  $\mu_B = 0.10$  expected annual log-return, and  $\sigma_B = 0.20$  standard deviation,
- A positive correlation of  $\rho = \rho_{AB} = 0.30$  between this two log-returns.

The value of the portfolio in one year will be

$$V_\pi(1) = \pi_1 S_A(0)e^{X_A} + \pi_2 S_B(0)e^{X_B}.$$

Compute through simulation the VaR for  $\alpha = 0.95$  and  $\alpha = 0.99$   $T = 1$  year, assuming  $S_A(0) = S_B(0) = 1$ , and

- $\pi_A = 0.5$  and  $\pi_B = 0.5$ .
- $\pi_A = -0.5$  and  $\pi_B = 1.5$ .
- $\pi_A = -1$  and  $\pi_B = 2$ .

Run 50,000 simulations for each computation.

7. (a) If  $X \sim \mathcal{N}(\delta, \gamma^2)$ , we say that  $Z = \exp(X)$  is a lognormal random variable. Compute the expectation and the variance of a lognormal random variable with parameters  $(\delta, \gamma^2)$

(b) Assume that two portfolios have the same expected returns and the same variances, but the first has normally distributed returns, while the second has lognormally distributed returns. According to their VaR, which is more risky to hold?

8. [Mean Returns and Standard Deviations] This exercise provides an idea about usual values appearing in practice normal and lognormal VaR calculations. The following table<sup>1</sup> shows the 1926–1985 yearly averages of US market, and the HSI values computed for the 1986–1996 period.

Security	Interest rate	Standard deviation
HSI	22.3%	33.3%
Common stock	12%	21.2%
Corporate bonds	5.1%	8.3%
Government bonds	4.4%	8.2%
Treasury bills	3.5%	3.4%

(a) Produce a table of the corresponding compound or log-returns, using formulas (1) and (2) (see below).

(b) Produce a table with the corresponding log-returns for one day. Take into account that to [scale](#) expected returns from annual basis to a daily basis you should:

- Divide by T in order to get the mean daily log-returns,
- Divide by  $\sqrt{T}$  in order to get the mean daily standard deviation.
- $T = 250$  as this is the number of trading days in a year.

9. When comparing Normal versus Lognormal returns, usually one says that

- $\mu$  and  $\sigma$ , the parameters of the normal returns, are the [discrete](#) expected return and standard deviation,

---

<sup>1</sup>Taken from: Shiryaev A.N. Essentials of Stochastic Finance, World Scientific 1999, pages 51,55

- $\mu$  and  $\sigma$ , the parameters of the lognormal returns, are the **continuous** expected return and standard deviation.

Prove the following **passage formulas**:

$$1 + \mu = \exp(\delta + \gamma^2/2), \quad (1)$$

$$\sigma = (1 + \mu)\sqrt{\exp(\gamma^2) - 1}. \quad (2)$$

assuming that

- $Z \sim \mathcal{N}(\delta, \gamma^2)$ ,
- $X \sim \mathcal{N}(\mu, \sigma^2)$ .
- $\mathbf{E}(1 + X) = \mathbf{E} e^Z$ ,
- $\mathbf{var}(1 + X) = \mathbf{var} e^Z$ .

You can use the moment generating function for  $Z$ :

$$\mathbf{E} \exp(tZ) = \exp(t\delta + t^2\gamma^2/2) \quad (\text{for all } t). \quad (3)$$

(b) Prove the formula (3). (Hint: Represent  $Z = \delta + \gamma\mathcal{N}$ , with  $\mathcal{N} \sim \mathcal{N}(0, 1)$ .)