

Statistical Methods and Calibration in Finance and Insurance

MA6622

Assignment of Lectures 1,2,3

Some of the questions posed can be answered finding information on the web. As this source is not always totally trustable, try to verify the information, for instance, in two independent web sites.

Exercises 5 and 6 are part of the assessment (Deadline: Friday 16th June).

We denote by $r(\infty)$ the continuous interest rate, and by $r(m)$ the simple interest rate that some deposit pays m times a year.

1. Suppose that a savings account doubles your capital in 10 years. Determine the continuous interest rate $r(\infty)$. Determine the simple interest rates $r(1)$ and $r(2)$.
2. Find out the interest rate of a bank account (in HKD) for 30, 60, 180 and 360 days. Compute the corresponding continuous interest rates for each case.

The following exercise, although is not related to finance, uses the notion of [arbitrage](#).

3. You have to friends, Oliver, from Germany, and Ronaldo, from Brazil. In Germany people are willing to bet 5:5 that Germany will win the FIFA 2006 World Cup (i.e. they bet 5 to receive 10 if Germany win). In Brazil, people bet 6:4 to Brazil (i.e. they bet 6 to receive 10 if Brazil win). Propose your friends a strategy to make a [free lunch](#), i.e. to win money without risk.
4. Find:
 - The difference between a [warrant](#) and an [option](#).
 - What does [open interest](#) mean in an options context.

The following exercise are related to the [Hang Seng Index \(HSI\)](#).

5. Find out what is the Hang Seng Index (HSI), the company names that constitute it, the corresponding weighting of each company, and the respective four groups these companies conform.
6. Find the corresponding data, to plot and print the following graphics:
 - The last 30 daily prices

- The last 60 daily prices
- The last 30 weekly prices.

Present the corresponding data in tables, explaining the corresponding sources, and type of prices (closing, means, etc.)

7. Remember that two probabilities \mathbf{P} and \mathbf{Q} are **equivalent** when $\mathbf{P}(A) = 0$ if and only if $\mathbf{Q}(A) = 0$. Exhibit a pair of probabilities \mathbf{P} and \mathbf{Q} in \mathbb{R} that are not equivalent.

The following exercise, related to martingales and filtrations is optional.

- 8.** (a) Prove that if $\{X_n\}$ is a martingale, then $\mathbf{E} X_n = \mathbf{E} X_0$.
- (b) We say that $\{x_n\}$ is a **martingale-difference** when $X_n = x_1 + \dots + x_n$, is a martingale. Prove that, in this case, $\mathbf{E} x_n = 0$. Prove that $\mathbf{E}(x_n x_{n+k}) = 0$, i.e. a martingale difference is an **uncorrelated** sequence.
- (c) What condition should satisfy a sequence of *independent* random variables with finite expectation to be a martingale-difference?