Optimal Stopping for a Diffusion with Jumps *

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Abstract

In this paper we give the closed form solution of some optimal stopping problems for processes derived from a diffusion with jumps. Within the possible applications, the results can be interpreted as pricing perpetual American Options under diffusion-jump information.

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1 Introduction

Let be given on a stochastic basis $(\Omega, \mathcal{F}, \mathbf{F}, P)$ a Wiener process $W = (W_t)_{t\geq 0}$, a Poisson process $N = (N_t)_{t\geq 0}$ with intensity $\lambda > 0$, and a sequence of independent nonnegative random variables $Y = (Y_k)_{k\geq 1}$, with identical distribution F. We will denote $F \sim \exp(\alpha)$ when F is exponential with parameter $\alpha > 0$. Assume that the processes W, N and Y are independent, and that the filtration $\mathbf{F} = (\mathcal{F}_t)_{t\geq 0}$ is the minimal filtration satisfying the usual conditions

***Running title:* Stopping a Diffusion with Jumps (see Jacod and Shiryaev (1987), page 2) such that the process $U = (U_t)_{t\geq 0}$ or $D = (D_t)_{t\geq 0}$ given by

$$U_t = x + \sigma W_t + \sum_{k=1}^{N_t} Y_k - at, \qquad t \geq 0, \tag{1}$$

and

$$D_t = x + \sigma W_t - \sum_{k=1}^{N_t} Y_k + at, \qquad t \geq 0, \tag{2}$$

are adapted to $\mathbf{F}$. U stands for jumps up, and D for jumps down. Here x, σ, and a are real constants with σ and a positive. τ is a stopping time (or stopping rule) relative to $\mathbf{F}$, if

$$\tau\colon \Omega \to [0, +\infty] \quad \text{and} \quad \{\tau \leq t\} \in \mathcal{F}_t \quad \text{for all} \quad t \geq 0.$$

Denote by $\mathcal{M}$ the class of all stopping times.

Consider the following problem: given a Borel function $g\colon \mathbf{R} \to \mathbf{R}$ and a random process $S = (S_t)_{t\geq 0}$ on $(\Omega, \mathcal{F}, \mathbf{F}, P)$ find a real function $s\colon \mathbf{R} \to \mathbf{R}$ and a stopping rule τ^*, such that

$$s(x) = \sup_{\tau \in \mathcal{M}} E(g(S_\tau)) = E(g(S_{\tau^*})). \tag{3}$$

Here s is called the cost function, and τ^* (the stopping time that realizes the supremum) the optimal stopping rule.

In the present paper we give the closed form solution to the problem (3) in the following four cases. The function g is given either by

$$g(x) = (e^x - K)^+, \quad \text{or} \quad g(x) = (K - e^x)^+,$$

with K a positive constant, and the process S is either U or D.

The results presented are generalizations of Mc Kean (1965) and Mordecki (1997), for pure diffusion and pure jumps respectively, and one of the four problems (Theorem 3) was solved by Zhang (1995). The problems considered are motivated by option pricing of perpetual American Options, and this kind of models were introduced by Merton (1990). For option pricing see also Karatzas (1988) or Shiryaev et al. (1994). Other applications of the presented results, for instance to investment under uncertainty can be found in Dixit and Pindyck (1994).

The paper is organized as follows. In section 2 we present the main results. The proofs, given in section 4 are based on a general Lemma, given in section 3. In section 5 a conclusion is presented.

2 Main Results

Theorem 1 *Let* U *be given by (1), and* $g(x) = (e^x - K)^+$. *Assume that* $F \sim \exp(\alpha)$, $\alpha > 1$ *and*

$$\frac{\sigma^2}{2} + \frac{\lambda}{\alpha - 1} < a. \tag{4}$$

Denote

$$x_0 = \log\left(K \frac{\lambda/\alpha - a}{\sigma^2/2 - a + \lambda/(\alpha - 1)}\right), \tag{5}$$

$$Q(p) = \frac{\sigma^2}{2}p^2 + \left(\frac{\alpha\sigma^2}{2} + a\right)p + \alpha a - \lambda. \tag{6}$$

Then, the solution to the optimal stopping problem (3) is

$$\begin{aligned} \tau^* &= \inf\{t \geq 0 \colon U_t \geq x_0\}, \\ s(x) &= \begin{cases} Ae^{p_1(x_0 - x)} + Be^{p_2(x_0 - x)}, & \text{if } x \leq x_0, \\ e^x - K, & \text{if } x > x_0. \end{cases} \end{aligned} \tag{7}$$

where $p_1 < 0$ *and* $p_2 < 0$ *are the roots of the equation* $Q(p) = 0$, *and the coefficients* A *and* B *are*

$$A = \frac{e^{x_0}(1 + p_2) - p_2 K}{p_2 - p_1}, \qquad B = \frac{e^{x_0}(1 + p_1) - p_1 K}{p_1 - p_2}.$$

Theorem 2 *Let* U *be given by (1) and* $g(x) = (K - e^x)^+$. *Assume that* F *is arbitrary. Let* [1]

$$a < \lambda \int_0^{+\infty} y dF(y). \tag{8}$$

Let β *be the unique positive root of the equation*

$$\frac{\sigma^2}{2}\beta^2 + a\beta + \lambda \int_0^{+\infty} (e^{-\beta y} - 1) dF(y) = 0 \tag{9}$$

Denote

$$x_0 = \log\left(K \frac{\beta}{\beta + 1}\right).$$

[1] Conditions (8) and (11) are corrected from the published version

Then, the solution to the optimal stopping problem (3) is

$$\begin{aligned} \tau^* &= \inf\{t \geq 0 \colon U_t \leq x_0\}, \\ s(x) &= \begin{cases} K - e^x, & \text{if } x \leq x_0, \\ (K - e^{x_0}) \exp\{-\beta(x - x_0)\}, & \text{if } x > x_0. \end{cases} \end{aligned} \tag{10}$$

Theorem 3 *Let D be given by (2) and $g(x) = (e^x - K)^+$. Assume that F is arbitrary. Let*

$$\frac{\sigma^2}{2} + a + \lambda \int_0^{+\infty} (e^{-y} - 1) dF(y) < 0. \tag{11}$$

Let $\beta > 1$ be the unique positive root of the equation (8). Denote

$$x_0 = \log (K \frac{\beta}{\beta - 1}).$$

Then, the solution to the optimal stopping problem (3) is

$$\begin{aligned} \tau^* &= \inf\{t \geq 0 \colon D_t \geq x_0\}, \\ s(x) &= \begin{cases} (e^{x_0} - K) \exp\{\beta(x - x_0)\}, & \text{if } x < x_0, \\ e^x - K, & \text{if } x \geq x_0. \end{cases} \end{aligned} \tag{12}$$

Theorem 4 *Let D be given by (2), and $g(x) = (K - e^x)^+$. Assume that $F \sim \exp(\alpha)$, and*

$$\frac{\lambda}{\alpha + 1} < \frac{\sigma^2}{2} + a. \tag{13}$$

Denote

$$x_0 = \log (K \frac{a - \lambda/\alpha}{\sigma^2/2 + a - \lambda/(\alpha + 1)}). \tag{14}$$

Then, the solution to the optimal stopping problem (3) is given by

$$\begin{aligned} \tau^* &= \inf\{t \geq 0 \colon D_t \leq x_0\}, \\ s(x) &= \begin{cases} K - e^x, & \text{if } x < x_0, \\ Ae^{p_1(x - x_0)} + Be^{p_2(x - x_0)}, & \text{if } x \geq x_0. \end{cases} \end{aligned} \tag{15}$$

where $p_1 < 0$ and $p_2 < 0$ are the roots of $Q(p) = 0$, with $Q(p)$ given by (6) and the coefficients A and B are

$$A = \frac{e^{x_0}(1 - p_2) + p_2 K}{p_2 - p_1}, \qquad B = \frac{e^{x_0}(1 - p_1) + p_1 K}{p_1 - p_2}.$$

3 Some preliminary results

First observe that both processes U and D are of the form $X = (X_t)_{t\geq 0}$ with

$$X_t = x + \sigma W_t + \sum_{k=1}^{N_t} Z_k + at, \qquad t \geq 0, \tag{16}$$

where Z_k are i.i.d. random variables with support on the whole real line and $a \in \mathbf{R}$.

In order to apply Itô's formula to the process $(s(X_t))_{t\geq 0}$ with s defined by (7), (10), (12) and (15) it is enough to observe that, although s is not $\mathbf{C}^2(\mathbf{R})$, $s''(x)$ is continuous for $x \neq x_0$, and has finite lateral limits. This gives, denoting $\mu(da)$ the signed measure (when restricted to compacts) which is the second derivative of s in the generalized sense,

$$\mu(da) = s''(a)da.$$

So Meyer-Itô formula applies (Theorem IV.51 of Protter (1992)) and a minor modifications in Corollary 1 to the same formula, necessary because s is only bounded on compacts, gives

$$s(X_t) - s(x) = \int_0^t s'(X_{s-})dX_s + \frac{1}{2}\int_0^t s''(X_{s-})d\left\langle X, X\right\rangle_s$$
$$+ \sum_{0\leq s\leq t} (s(X_s) - s(X_{s-}) - s'(X_{s-})\Delta X_s).$$

Furthermore, in our case, denoting $X^c = (X_t^c)_{t\geq 0}$, $X^d = (X_t^d)_{t\geq 0}$, with

$$X_t^c = x + \sigma W_t + at, \qquad X_t^d = \sum_{i=1}^{N_t} Z_i,$$

we have

$$\int_0^t s'(X_{s-})dX_s + \sum_{0\leq s\leq t} (s(X_s) - s(X_{s-}) - s'(X_{s-})\Delta X_s) =$$

$$\int_0^t s'(X_{s-})dX_s^c + \int_0^t \int_{\mathbf{R}} [s(X_{s-} + x) - s(X_{s-})] * (\mu(\omega, dx, ds) - \nu(dx, ds))$$
$$+ \int_0^t \int_{\mathbf{R}} [s(X_{s-} + x) - s(X_{s-})] * \nu(dx, ds),$$

where $\mu = \mu(\omega, dx, ds)$ is the jump measure corresponding to X^d, and

$$\nu = \nu(dx, dt) = \lambda dt F(dx)$$

its compensator.

So, Itô's formula in our case reads

$$s(X_t) - s(x) = \int_0^t (Ls)(X_{s-})ds + M(s)_t \tag{17}$$

with

$$(L^X s)(x) = \frac{1}{2}\sigma^2 s''(x) + as'(x) + \lambda \int_{\mathbf{R}} (s(x+y) - s(x))dF(x)$$

the infinitesimal generator of the process X and the local martingale $M(s) = (M(s)_t)_{t\geq 0}$ given by

$$M(s)_t = \sigma \int_0^t s'(X_{s-})dW_s + \int_0^t \int_{\mathbf{R}} (s(X_{s-} + x) - s(X_{s-})) * (\mu - \nu).$$

Lemma 1 *Let $X = (X_t)_{t\geq 0}$ be given by (16). Let s and g be real Borel functions, such that s is convex with s'' continuous for $x \neq x_0$, for some $x_0 \in \mathbf{R}$, and has finite lateral limits. C^*, the continuation region is an open halfline of the form $(-\infty, x_0)$ or $(x_0, +\infty)$*

Let

$$\tau^* = \inf\{t \geq 0 \colon X_t \notin C^*\}.$$

Assume that the following five conditions hold:

(i) $(Ls)(x) = 0, \qquad \forall x \in C^*.$

(ii) $(Ls)(x) \leq 0, \qquad \forall x \neq x_0.$

(iii) $0 \leq g(x) \leq s(x), \qquad \forall x \in \mathbf{R}.$

(iv) $s(X_{\tau^* \wedge T \wedge t}) \leq Z$, *$P$-a.s for all $T \in \mathcal{M}$ and for all $t \in \mathbf{R}^+$, with Z an integrable random variable, that is $E|Z| < +\infty$.*

(v) $s(X_{\tau^*}) = g(X_{\tau^*})$, *$P$-a.s.*

Then, under these assumptions, the pair (τ^*, s) is the solution for the optimal stopping problem for the function g and the process X, that is:

$$s(x) = \sup_{\tau \in \mathcal{M}} E(g(X_\tau)) = E(g(X_{\tau^*})).$$

Proof. By (17) we have

$$s(X_t) - s(x) = \int_0^t (Ls)(X_{s-})ds + M(s)_t.$$

We have to prove assertions (a) and (b):

(a) $s(x) = Eg(X_{\tau^*})$,

(b) $s(x) \geq Eg(X_\sigma) \ \ \forall \sigma \in \mathcal{M}$.

By conditions (v) and (iii) in our hypothesis this is equivalent to proving

(a′) $s(x) = Es(X_{\tau^*})$,

(b′) $s(x) \geq Es(X_\sigma) \ \ \forall \sigma \in \mathcal{M}$.

Taking into account that $A_t = -\int_0^t (Ls)(X_{s-})ds$ is increasing (by condition (ii)) and $A_{\tau^*} = 0$ (by condition (i)) we have to verify

(a″) $E(M_{\tau^*}) = 0$,

(b″) $E(M_\sigma) \leq 0 \ \ \forall \sigma \in \mathcal{M}$.

As $M(s) = (M(s)_t)_{t \geq 0}$ is a local martingale with $M(s)_0 = 0$, for a localizing sequence (τ_n) we have

$$E(M(s)_{t \wedge \tau^* \wedge \tau_n}) = E(M(s)_0) = 0.$$

As

$$-s(x) \leq M(s)_{t \wedge \tau^* \wedge \tau_n} \leq s(X_{t \wedge \tau^* \wedge \tau_n}) \leq Z$$

(a″) follows by dominated convergence.

As $M(s)_t \geq -s(x)$, by Fatou's Lemma the local martingale $M(s)$ is in fact a supermartingale with $EM(s)_0 = 0$ and (b″) follows.

4 Proof of the theorems

In view of the previous Lemma, the proofs of Theorems 1 to 4 reduces to the verification of conditions (i) to (v) in each case. As proofs of Theorems 1 and 4, and 2 and 3 are similar respectively, we give proofs of Theorems 1 and 2 in detail and only give a sketch of the proofs of Theorems 3 and 4.

Proof of Theorem 1. The infinitesimal generator of the process U is

$$(L^U f)(x) = \frac{1}{2}\sigma^2 f''(x) - af'(x) + \lambda \int_0^{+\infty} (f(x+y) - f(x))dF(y).$$

(i) For $x < x_0$ and $s(x)$ as in (7) we have

$$(L^U s)(x) = \frac{1}{2}\sigma^2 s''(x) - as'(x) + \lambda \int_0^{+\infty} [s(x+y) - s(x)]\alpha e^{-\alpha y} dy$$

$$= Ae^{p_1(x_0-x)}(\frac{1}{2}\sigma^2 p_1^2 + ap_1 - \lambda\frac{p_1}{p_1+\alpha}) + Be^{p_2(x_0-x)}(\frac{1}{2}\sigma^2 p_2^2 + ap_2 - \lambda\frac{p_2}{p_2+\alpha})$$

$$-\lambda\alpha e^{-\alpha(x_0-x)}(\frac{A}{p_1+\alpha} + \frac{B}{p_2+\alpha} - \frac{e^{x_0}}{\alpha-1} + \frac{K}{\alpha})$$

$$= A\frac{p_1}{p_1+\alpha}e^{p_1(x_0-x)}Q(p_1) + B\frac{p_2}{p_2+\alpha}e^{p_2(x_0-x)}Q(p_2)$$

$$-\lambda\alpha e^{-\alpha(x_0-x)}(\frac{A}{p_1+\alpha} + \frac{B}{p_2+\alpha} - \frac{e^{x_0}}{\alpha-1} + \frac{K}{\alpha}).$$

We know $Q(p_1) = Q(p_2) = 0$ (see (6)).

$$\frac{A}{p_1+\alpha} + \frac{B}{p_2+\alpha} - \frac{e^{x_0}}{\alpha-1} + \frac{K}{\alpha} = 0$$

is verified taking into account that

$$\begin{aligned} (p_1+\alpha)(p_2+\alpha) &= -\tfrac{2\lambda}{\sigma^2} \\ p_2 A + p_1 B &= (p_1+p_2)(e^{x_0} - K) + e^{x_0} \end{aligned} \qquad (18)$$

and (5).

(ii) We have to see that for $x > x_0$, $L^U s(x) \le 0$.

$$(L^U s)(x) = \frac{\sigma^2}{2}e^x - ae^x + \lambda \int_0^{+\infty} e^x(e^y - 1)dF(y) = e^x(\frac{\sigma^2}{2} - a + \frac{\lambda}{\alpha-1}) < 0$$

in accordance with (4).

(iii) $s(x) = g(x)$ if $x > x_0$. For $x \leq x_0$ consider the auxiliar real function

$$a(x) = s(x) - g(x) = Ae^{p_1(x_0-x)} + Be^{p_2(x_0-x)} - e^x + K$$

We have to see $a(x) \geq 0$ for all $x \leq x_0$. We know that $a(x_0) = 0$, $a'(x_0) = 0$ and $\lim_{x\to-\infty} a(x) = K$. If $a(x_1) = 0$ with $-\infty < x_1 < x_0$ then $a(x) = 0$ for all $x \in (-\infty, x_0]$ by the uniqueness Theorem for Differential Equations because $a(x)$ is the solution of a third order linear differential equation.

(iv) Take $Z = s(U_{\tau^*}) + x_0$. Then, $s(U_{\tau^*\wedge T\wedge t}) \leq Z$ follows from the fact that on the set $\{\tau^* < +\infty\}$ we have $U_{\tau^*\wedge T\wedge t} \leq U_{\tau^*}$ and the function s is increasing. On the set $\{\tau^* = +\infty\}$ we have $U_{\tau^*\wedge T\wedge t} \leq x_0$.

Observe now, that

$$s(U_{\tau^*}) \leq (e^{U_{\tau^*}} - K)^+ \leq \exp\{\sup_{0\leq t\leq+\infty} U_t\}$$

and by Mordecki (1997) we have $E(\exp\{\sup_{0\leq t\leq+\infty} U_t\}) < \infty$.

(v) We have

$$\lim_{x\to-\infty} s(x) = \lim_{x\to-\infty} g(x) = 0$$

so $s(U_{\tau^*}) = g(U_{\tau^*}) = 0$ on $\{\tau^* = +\infty\}$ because $\lim_{t\to+\infty} U_t = -\infty \quad P-a.s.$, and $s(x) = g(x)$ if $x \geq x_0$, so $s(U_{\tau^*}) = g(U_{\tau^*})$ on $\{\tau^* < +\infty\}$, concluding the proof of Theorem 1.

Proof of Theorem 2.

(i) If $s(x) = (K - e^{x_0})e^{-\beta(x-x_0)}$ for $x > x_0$

$$(L^U s)(x) = (K - e^{x_0})e^{-\beta(x-x_0)}(\frac{\sigma^2}{2}\beta^2 + a\beta + \lambda\int_0^{+\infty}[e^{-\beta y} - 1]dF(y)) = 0$$

by (9).

(ii)[2]. In this case, for $x < x_0$

$$(L^U s)(x) = e^x[-\frac{1}{2}\sigma^2 + a + \lambda\int_0^{+\infty}(s(x+y) - s(x))e^{-x}dF(y)]$$

$$\leq e^x[-\frac{1}{2}\sigma^2 + a + \lambda\int_0^{+\infty}(s(x_0+y) - s(x_0))e^{-x_0}dF(y)]$$

[2]We complete some missing arguments in the published version

$$\leq e^{x-x_0}[\frac{1}{2}\sigma^2 s''(x_0+) + as'(x_0) + \lambda \int_0^{+\infty} (s(x_0+y) - s(x_0))dF(y)] = 0$$

because $(s(x+y)-s(x))e^{-x} \leq (s(x_0+y)-s(x_0))e^{-x_0}$ by convexity of $s(\log z)$ in z, and $s''(x_0-) < s''(x_0+)$.

(iii) For $x > \log K$, $g(x) = 0$. When $x_0 < x < \log K$ the argument follows as in the proof of (iii) in Theorem 1, considering the auxiliar function $a(x) = s(x) - g(x)$ that verifies $a(x_0) = a'(x_0) = 0$, $a(\log K) > 0$, and is the solution of a second order linear differential equation.

(iv) is inmediate, because in this case we have

$$0 \leq s(U_t) \leq x_0, \quad \text{for all} \quad t \geq 0.$$

(v) In this case we have $U_{\tau^*} = x_0$ on the set $\{\tau^* < +\infty\}$, and in consequence $g(U_{\tau^*}) = s(U_{\tau^*}) = s(x_0)$ on $\{\tau^* < +\infty\}$. As $\lim_{x\to+\infty} g(x) = \lim_{x\to+\infty} s(x) = 0$ the result follows.

Proof of Theorem 3.

For the process D defined in (2)

$$(L^D f)(x) = \frac{1}{2}\sigma^2 f''(x) + af'(x) + \lambda \int_0^{+\infty} (f(x-y) - f(x))dF(x)$$

(i) If $s(x) = (e^{x_0} - K)e^{\beta(x-x_0)}$ for $x < x_0$

$$(L^D s)(x) = (e^{x_0} - K)e^{\beta(x-x_0)}(\frac{\sigma^2}{2}\beta^2 + a\beta + \lambda \int_0^{+\infty} [e^{-\beta y} - 1]dF(y)) = 0$$

by the election of β.

(ii)[3] In this case, for $x \geq x_0$

$$(L^D s)(x) = e^x[\frac{1}{2}\sigma^2 + a + \lambda \int_0^{+\infty} (s(x-y) - s(x))e^{-x}dF(y)]$$

$$\leq e^x[\frac{1}{2}\sigma^2 + a + \lambda \int_0^{+\infty} (s(x_0-y) - s(x_0))e^{-x_0}dF(y)]$$

$$\leq e^{x-x_0}[\frac{1}{2}\sigma^2 s''(x_0-) + as'(x_0) + \lambda \int_0^{+\infty} (s(x_0-y) - s(x_0))dF(y)] = 0$$

because $(s(x-y)-s(x))e^{-x} \leq (s(x_0-y)-s(x_0))e^{-x_0}$ by convexity of $s(\log z)$ in z, and $s''(x_0+) < s''(x_0-)$.

[3] Idem pg. 9

(iii) It is similar to (iii) in Theorem 1. Now the auxiliar function is

$$a(x) = (e^{x_0} - K)e^{\beta(x-x_0)} - e^x + K \quad \text{for} \quad x \leq x_0$$

that verifies $a(x_0) = a'(x_0) = 0$ and $\lim_{x\to-\infty} = K$

(iv) follows word by word as (iv) Theorem 1.

(v) follows as (v) in Theorem 2.

Proof of Theorem 4.

(i) is similar to (i) in Theorem 1, but the necessary relations are now (14), (18), and

$$p_2 A + p_1 B = (p_1 + p_2)(K - e^{x_0}) + e^{x_0}.$$

(ii) For $x \leq x_0$ we have

$$(L^D s)(x) = e^x\Big(-\frac{1}{2}\sigma^2 - a + \lambda \int_0^{+\infty} (1 - e^{-y})\alpha \exp\{-\alpha y\} dy\Big)$$

$$= e^x\Big(-\frac{1}{2}\sigma^2 - a + \frac{\lambda}{\alpha + 1}\Big) < 0$$

by (13).

(iii) follows as (iii) in Theorem 2.

(iv) is inmediate, because

$$0 \leq s(X_t) \leq K \qquad P - a.s.$$

(v) follows as (v) in Theorem 1.

5 Conclusion

The paper presented contains some closed form solutions to optimal stopping problems when the stopped process has jumps. As the main application is option pricing when returns are allowed to jump at poissonian times, and alternative title would have been “Pricing Perpetual American Options under Jump-Diffusion information”. The results presented are in the spirit of Shiryaev (1973) and Mc Kean (1965), though a modification in the “smooth pasting” principle was necessary to obtain the results (See Lemma 1 in section 3). The results can be used to price perpetual American Options for assets with jumps, once a reference measure in the incomplete market is choosen, as suggested by Föllmer and Scwheizer (1990).

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