

Noncommutative structures on spaces of modular forms

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For any subgroup Γ of $SL(2, \mathbb{C})$ acting by homography on a commutative algebra R of functions in one complex variable z , we consider for any integer k the \mathbb{C} -vector space $M_k(\Gamma, R)$ of weight k modular forms. The action of Γ on R can be canonically extended into a one parameter family of actions of Γ on the noncommutative algebra $B = R((x; -\partial_z))$ of pseudodifferential operators in one variable x with coefficients in R and multiplication twisted by the derivative related to z . A theorem of P. Cohen, Y. Manin and Don Zagier proves that the invariant spaces B^Γ are isomorphic to some interesting space $\mathcal{M} = \bigcup_{j \in \mathbb{Z}} \prod_{k \geq j} M_{2k}(\Gamma, R)$ of sequences of even weight modular forms. We present in this talk (from a joint work with F. Martin) an algebraic study of the algebras B^Γ , and some applications to the associated noncommutative algebra structures on \mathcal{M} .