The spin limit for cosmological black holes

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The area-angular momentum inequality for black holes in cosmological spacetimes
María Eugenia Gabach Clément, Martín Reiris and Walter Simon 2015 Class. Quantum Grav. ***32*** *145006*
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— María Eugenia Gabach Clément

In colloquial terms, the main achievement of our *recent CQG article* is simple to state: We have proven that the angular momentum J of an axially symmetric black hole (the Noether current) with surface area A satisfies the bound.

$$|J| \leq \frac{A}{8\pi}\sqrt{\left(1 - \frac{\Lambda A}{4\pi}\right)\left(1 - \frac{\Lambda A}{12\pi}\right)}.$$

Here Λ is the cosmological constant – a standard ingredient in Einstein's equations which adopts the small positive value $\Lambda \approx 10^{-52} m^2$
in the currently favoured models of the universe.

Equality in the above bound characterizes the one-parameter family of so-called "extreme Kerr-deSitter" geometries. These are limiting cases of the two-parameter Kerr-deSitter family which are, in the cosmological setting, the only known metrics which represent black holes rotating stationarily (i.e. at constant speed). Moreover, by pure algebra, the above bound implies a universal bound $|J|\Lambda < 0.1703...$ on the angular momentum of black holes, which is saturated for one particular extreme Kerr-deSitter configuration.

Since the root in the above bound is less than one, our result strengthens its key predecessor, namely $8\pi|J| \leq A$, which is sharp for extreme Kerr. As the cosmological expansion tends to "rip apart everything", it is not too surprising that it also destabilises spinning black holes, and hence reduces the allowed angular momentum. On the other hand, but even less surprising: The influence of Λ on this limit is a tiny one. Natural examples to illustrate this are black hole partners of binaries, whose spin can be inferred from the inspiralling material. Observations indicate that such black holes can approach the extreme Kerr (de-Sitter) limit very closely, in fact up a few percent, or up to observational uncertanities. However, the observed and already "baptized" holes (such as GRS 1915+105) have only a few solar masses and hence areas $A \approx 10^9 m^2$, which is negligible compared to the value of Λ^{-1} given above. The same applies even to the largest black hole near the centre of our galaxy for which $A \approx 10^{21} m^2$, while its spin is unknown. In fact the root in our inequality will only become relevant for hypothetical giant black holes of at least the size of our whole galaxy. Moreover, if two such giant holes approached each other with spins aligned, they cannot merge if the sum of their angular momenta were to violate our universal bound on $|J|\Lambda$.

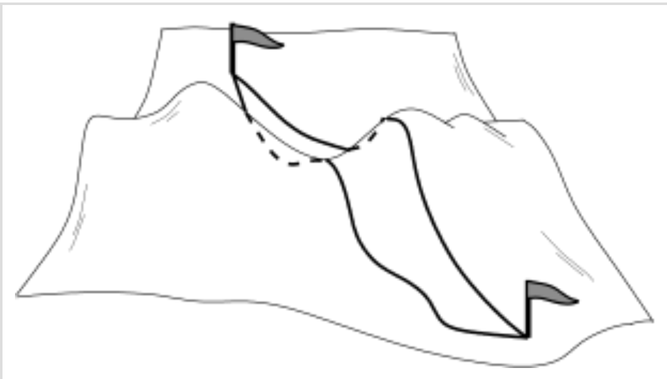
— Martín Reiris

These considerations suggest that our results are not relevant in astrophysics – at least when Λ is interpreted as cosmological constant (we indicate a different interpretation below). In any case, we believe that the main value of our article lies in its mathematical aspects. We therefore continue stating precisely what we proved, and illustrate the key steps and methods of the proof.

Our "black holes" are in fact "stable marginally outer trapped surfaces" (MOTS) – a natural and weak requirement, which only restricts the expansions of the "outgoing" family of orthogonally emanting null geodesics on and near the MOTS. (The present notion of stability is unrelated to dynamical stability). The reason why we prove a bound on the surface area rather than on the mass of the MOTS (which at first sight seems easier to determine) is that the latter is mathematically just not well-defined in the present (non-stationary, non-asymptotically flat) setting. Energy near the MOTS is allowed as long as it satisfies the dominant energy condition. The proof consists of two main steps: The first one makes use of the stability property of the MOTS to bound its area from below in terms of a so-called "mass functional" $\mathcal{M}$. Compared to our "role model" namely the proof of $8\pi|J| \leq A$, we now face the problem that $\mathcal{M}$ depends on A itself. We therefore modify $\mathcal{M}$ by "freezing" A and J to values corresponding to a certain extreme configuration, and by adapting the dynamical variables in $\mathcal{M}$ suitably. The second step consists of showing existence and uniqueness of a minimiser of $\mathcal{M}$. We first prove that every critical point

— Walter Simon

of $\mathcal{M}$ is a local minimum. It is at this point that Λ finally cooperates, as it allows to establish the required coercivity property of $\mathcal{M}$. But reasonably well-behaved functionals on compact spaces without boundary either have a unique local (and hence global) minimum, or there is at least one local maximum as well – a fact which is intuitively clear from the following figure



and whose rigorous formulation is called "mountain pass theorem". In the present situation the first alternative applies, and the unique minimum corresponds to the horizon geometry of extreme Kerr-deSitter spacetimes.

We conclude with mentioning three possible applications and generalisations of our result. Firstly, we note that the above proof is completely insensitive to the value of Λ, while a term of "Λ-form" also arises in our equations in the presence of matter with density $\rho \geq \Lambda/(8\pi)$. This allows an application of our area bound to black holes which might form inside stars, in particular inside dense and rapidly spinning objects like millisecond pulsars. Secondly, we recall from another CQG paper that the bound $A \geq 8\pi|J|$ has been strengthened to a form including electric and magnetic charges. Here an inclusion of Λ seems natural and possible. We note in this context that black holes may carry any type of charge as a purely "topological" effect, i.e. without containing any charged matter. Our last but not least remark concerns binary-black hole-collisions and their numerical simulations. In this business it is key to understand the distribution of energy and angular momentum between the emitted gravitational waves on the one hand, and the final black hole on the other hand. However, except for "head-on collisions" with aligned spins, this is clearly a non-axially symmetric setting. Applying area-angular momentum inequalities here is interesting, but this first requires a corresponding generalised notion of angular momentum. A simulation of a collision of nearly extreme holes involving such an adapted $\mathcal{J}$ has been obtained recently (see the paper in CQG and the corresponding CQG+ entry). As in all cosmological applications of our inequality, we cannot expect the tiny cosmological constant to produce any observable effects here either.

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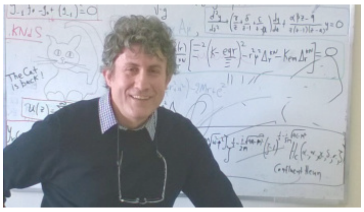

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