Partial hyperbolicity and attracting regions in 3-dimensional manifolds

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Thesis defense

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- Study of attracting regions (semilocal study).
- Understand global robust dynamical behaviour.

Attractors and attracting sets

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These examples pose a number of questions:

- Structure of quasi-attractors.

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Higher dimensions: Bonatti-Li-Yang provided the first examples of dynamics without attractors in a robust fashion (C^r-generically in C^r-open sets).

These examples pose a number of questions:

- Structure of quasi-attractors.
- Basin problem, ergodic attractors, Milnor attractors.

Definitions: Decomposition of the dynamics.

Conley's theory: $f : X \to X$ homeomorphism of a compact metric space, $\exists \varphi : X \to \mathbb{R}$ such that:

- $\varphi(f(x)) \leq \varphi(x)$ for every x.
- $\varphi(f(x)) = x$ if and only if x is *chain-recurrent*.
- Each chain-recurrence class attains different values for φ. The image of the chain-recurrence set has empty interior.

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x is *chain-recurrent* if $\forall \varepsilon$ exists ε-pseudo-orbit from x to x: i.e. $x = z_0, \dots z_k = x$ with $k \geq 1$ and $d(z_{i+1}, f(z_i)) < \varepsilon$.

The set of chain-recurrent points $CR(f)$ is partitioned into *chain-recurrence classes*: $x \sim y$ if $\forall \varepsilon$ there exists ε-pseudo-orbit from x to y and from y to x.

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A chain recurrence class Q is a *quasi-attractor* if it admits a basis of neighborhoods U_n such that $f(\overline{U_n}) \subset U_n$.

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Definition (Attractors)

A quasi-attractor is an *attractor* if it is isolated as chain-recurrence class.

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Dissipativeness is not clear.

Recall on invariant splittings

Definition

A compact f-invariant set Λ has a *dominated splitting* if $T_\Lambda M = E \oplus F$ is a Df-invariant splitting and $\exists N \geq 0$ s.t. $\forall x \in \Lambda$ and $\forall v_E \in E(x)$ and $\forall v_F \in F(x)$ unit vectors we have:

$$\|Df^N v_E\| < \frac{1}{2}\|Df^N v_F\|$$

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A Df-invariant bundle E is *uniformly contracting* (resp. *uniformly expanding*) if $\exists N > 0$:

$$\|Df^N|_E\| < \frac{1}{2} \qquad (\text{resp.} \|Df^{-N}|_E\| < \frac{1}{2})$$

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We overcome by using Gourmelon's Franks Lemma, Bonatti-Bochi's cocyle perturbation techniques and Lyapunov stability.

Bi-Lyapunov stable classes.

Corollary

For C^1-generic diffeomorphisms we know:

- *In dimension 2, if Q is a bi-Lyapunov stable homoclinic class then $Q = \mathbb{T}^2$ and f is Anosov.*
- *In dimension 3, a bi-Lyapunov stable homoclinic class has non-empty interior.*
- *In any dimension we know they admit some dominated splitting.*

Bi-Lyapunov classes for C^1-generic diffeomorphisms are classes which are both quasi-attractors and quasi-repellers.

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Question

Can a C^1-generic diffeomorphism have *countably* infinitely many chain-recurrence classes?

Would be opposed with concept of *Viral Dynamics*.

New examples

Theorem

There exists $\mathcal{U}$ *open in* $\mathrm{Diff}^r(\mathbb{T}^3)$ *(*$r \geq 1$*) such that there exists a residual subset* $\mathcal{G} \subset \mathcal{U}$ *such that:*

- *For every* $f \in \mathcal{U}$*,* f *has a unique quasi-attractor* Q *which is a Milnor attractor (and if* f *is* C^2 *it admits a unique SRB measure).*
- *For every* $f \in \mathcal{U}$ *if* $R \neq Q$ *is a chain-recurrence class,* R *is contained in a periodic normally expanding two-dimensional disk.*
- *For every* $f \in \mathcal{G}$ *the diffeomorphism* f *has no attractors (the quasi-attractor is not isolated).*

Milnor attractors-SRB measures

An SRB-*measure* for $f : M \to M$ is an invariant measure μ such that there exists a *positive Lebesgue measure subset* $B(\mu) \subset M$ such that for every continuous function $\varphi : M \to \mathbb{R}$ and $x \in B(\mu)$ we have:

$$\frac{1}{n} \sum_{i=0}^{n-1} \varphi(f^i(x)) \to \int \varphi d\mu$$

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Seems well suited for C^1-dynamics where we do not control distortion.

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- We can do this also on any manifold (of dimension ≥ 3) but we do not know about the SRB measure.
- The examples of Bonatti-Li-Yang are accumulated by classes which are not in disks (Bonatti-Shinohara).
- If Smale conjecture (dimension 2) is true one would obtain (generic) examples with countably many chain-recurrence classes.

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Key point: The boundary of the fibers of the semiconjugacy are contained in the quasi-attractor. Fibers are invariant under unstable holonomy.

Digression

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With C.Bonatti,S.Crovisier and N.Gourmelon we proved:

Theorem

There exist open sets of diffeomorphisms where isolated chain recurrence classes which are not robustly transitive (in any C^r-topology, $r \geq 1$).

Our examples are not quasi-attractors, it remains open whether it can be done for quasi-attractors.

Global robust dynamical behavior

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Generalizing results of Mañe in dimension 2 we have:

Theorem (Diaz-Pujals-Ures)

$f : M^3 \to M^3$ is robustly transitive then f is *partially hyperbolic*.

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Theorem (Diaz-Pujals-Ures)

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Two kinds of partial hyperbolicity: $T\mathbb{T}^3 = E^{cs} \oplus E^u$ and strong: $T\mathbb{T}^3 = E^s \oplus E^c \oplus E^u$.

Long term goal

Understand the relationship between:

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- Topological properties.

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(Pujals-Bonatti-Wilkinson) Proposal for classification of strong partially hyperbolic transitive diffeomorphisms in dimension 3.

The classification is *modulo center foliations*.

Integrability problem

Definition

Let $f : M \to M$ be *partially hyperbolic* (i.e. it has a dominated splitting $TM = E \oplus F$ with either E uniformly contracting or F uniformly expanding). We say that f is *dynamically coherent* if there exist f-invariant foliations $\mathcal{F}_E$ and $\mathcal{F}_F$ tangent respectively to E and F.

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Definition

A partially hyperbolic diffeomorphism f (with splitting $TM = E^{cs} \oplus E^u$) is *almost dynamically coherent* if there exists a foliation $\mathcal{F}$ transverse to E^u.

It is an open and closed property!

Strong partial hyperbolicity

In dimension 3.

Definition

A diffeomorphism $f : M^3 \to M^3$ is *strongly partially hyperbolic* (SPH) if there exist a dominated splitting

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into one-dimensional bundles. E^s is uniformly contracting and E^u is uniformly expanding.

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into one-dimensional bundles. E^s is uniformly contracting and E^u is uniformly expanding.

Dynamical coherence: All the subbundles ($E^s \oplus E^c$, $E^c \oplus E^u$ and E^c) are integrable to a f-invariant foliation.

Previous Results

Theorem (Brin-Burago-Ivanov)

Under a stronger (absolute) version of SPH, if $f : \mathbb{T}^3 \to \mathbb{T}^3$ *is SPH then it is dynamically coherent.*

This was used by Hammerlindl to get *leaf conjugacy*.

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Theorem (Rodriguez Hertz-Rodriguez Hertz-Ures)

There exists a (non transitive) SPH diffeomorphism in $\mathbb{T}^3$ which is NOT dynamically coherent.

Statement of results

Theorem

Let $f : \mathbb{T}^3 \to \mathbb{T}^3$ a PH diffeomorphism isotopic to Anosov and almost dynamically coherent. Then f is dynamically coherent.

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Let $f : \mathbb{T}^3 \to \mathbb{T}^3$ a SPH diffeomorphism.

- *Either there exists a repelling torus T tangent to $E^s \oplus E^u$ or,*
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Corollary

If $f : \mathbb{T}^3 \to \mathbb{T}^3$ is SPH and $\Omega(f) = \mathbb{T}^3$ then f is dynamically coherent.

Some results on foliations

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Classification in $\mathbb{T}^3$: We give a classification of foliations without torus leaves in $\mathbb{T}^3$.

Global product structure: We give a quantitative version of how small must the holonomy be with respect to the local product structure in order to get global product structure.

Results of Brin-Burago-Ivanov

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- (Burago-Ivanov) f is almost dynamically coherent.
- (Brin-Burago-Ivanov)If $M = \mathbb{T}^3$. $f_* : \mathbb{R}^3 \to \mathbb{R}^3$ is SPH (either f_* is hyperbolic or f_* "is" Anosov$\times Id_{S^1}$).

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    - The plane projects into a torus: We find a repelling torus.
    - The plane close to the center stable leaf is the center unstable plane: Estimate growth of diameter and apply Novikov's theorem.

Conclusions and Questions

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- Obtain dynamical consequences from the existence of foliations.

With Hammerlindl, we have obtained leaf conjugacy for SPH diffeos in $\mathbb{T}^3$ and nilmanifolds. We have advanced in the Solvemanifold case.

Thanks! Gracias! Merci!